Announcements

• Assignment 4 is due Nov 30, 11:59 PM
• Final exam is Dec 8, 7:00 PM-9:59 PM
• Please complete TA and course evaluations
Electromagnetic Spectrum

- Increasing Frequency ($\nu$)
- Increasing Wavelength ($\lambda$)

Visible spectrum

- $\nu$ (Hz)
- $\lambda$ (m)

- $\gamma$ rays
- X rays
- UV
- IR
- Microwave
- FM Radio waves
- AM
- Long radio waves

- $10^{-16}$
- $10^{-14}$
- $10^{-12}$
- $10^{-10}$
- $10^{-8}$
- $10^{-6}$
- $10^{-4}$
- $10^{-2}$
- $10^{0}$
- $10^{2}$
- $10^{4}$
- $10^{6}$
- $10^{8}$

- $400$
- $500$
- $600$
- $700$
The appearance of colors

• Color appearance is strongly affected by (at least):
  – spectrum of lighting striking the retina
  – other nearby colors (space)
  – adaptation to previous views (time)
  – “state of mind”
Separating visible light
4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, Brian Wandell, 1995, via B. Freeman slides
note: black & white
Light Spectrum
Talking about colors

1. Spectrum –
   • A positive function over interval 400nm-700nm
   • “Infinite” number of values needed.

2. Names
   • red, harvest gold, cyan, aquamarine, auburn, chestnut
   • A large, discrete set of color names

3. R,G,B values
   • Just 3 numbers
Color Reflectance

Measured color spectrum is a function of the spectrum of the illumination and reflectance

From Foundations of Vision, Brian Wandell, 1995, via B. Freeman slides
Illumination Spectra

Blue skylight

Tungsten bulb

From Foundations of Vision, Brian Wandell, 1995, via B. Freeman slides
Why is this important?

• The color measured by a camera depends on
  – The camera response function
  – Spectrum of illuminant
  – Reflectance properties of objects in the scene

• The “color” of an object does not actually make a lot of sense, unless you talk about it under a reference light
Measurements of relative spectral power of sunlight, made by J. Parkkinen and P. Silfsten. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm. The color names on the horizontal axis give the color names used for monochromatic light of the corresponding wavelength --- the “colors of the rainbow”. Mnemonic is “Richard of York got blisters in Venice”.

Violet  Indigo Blue  Green  Yellow  Orange  Red

- cloudless sky, sunset
- slightly cloudy, sun visible
- cloudless sky, bright snow
- cloudy, gray sky
- slightly cloudy, sun behind a cloud
Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E.Koivisto.
Fresnel Equation for Polished Copper

\[ \begin{align*}
F(r) &= \frac{2}{\pi} \int_{0}^{\pi/2} \frac{1}{r} \sqrt{1 - \frac{r^2}{\lambda^2}} \, d\theta \\
\end{align*} \]
RGB Color Model

- Based on human perception of color

Red cones
Green cones
Blue cones

3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.
Color receptors

“Red” cone  “Green” cone  “Blue” cone

Response of $k$th cone = $\int \rho_k(\lambda)E(\lambda)d\lambda$
Three types of cones: R, G, B

Response of $k$th cone = $\int \rho_k(\lambda) E(\lambda) d\lambda$

There are three types of cones
- S: Short wave lengths (Blue)
- M: Mid wave lengths (Green)
- L: Long wave lengths (Red)

- Three attributes to a color
- Three numbers to describe a color
Color matching experiment

4.10 THE COLOR-MATCHING EXPERIMENT. The observer views a bipartite field and adjusts the intensities of the three primary lights to match the appearance of the test light. (A) A top view of the experimental apparatus. (B) The appearance of the stimuli to the observer. After Judd and Wyszecki, 1975.

Not on a computer Screen
Color Matching

Not on a computer Screen
Color matching experiment 1
Color matching experiment 1

\[ p_1 \quad p_2 \quad p_3 \]
Color matching experiment 1
Color matching experiment 1

The primary color amounts needed for a match

$\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3$
Color matching experiment 2
Color matching experiment 2
Color matching experiment 2
Color matching experiment 2

We say a "negative" amount of $p_2$ was needed to make the match, because we added it to the test color’s side.

The primary color amounts needed for a match:

$p_1$ $p_2$ $p_3$

slide from T. Darrel
The principle of trichromacy

- Experimental facts:
  - Three primaries will work for most people if we allow subtractive matching
    - Exceptional people can match with two or only one primary.
    - This could be caused by a variety of deficiencies.
  - Most people make the same matches.
    - There are some anomalous trichromats, who use three primaries but make different combinations to match.
Color matching functions

• Choose primaries, say $p_1(\lambda)$, $p_2(\lambda)$, $p_3(\lambda)$

• For monochromatic (single wavelength) energy function, what amounts of primaries will match it?

• That is, for each wavelength $\lambda$, determine how much of $p_1$, $p_2$, and $p_3$ is needed to match light of that wavelength alone

• These are color matching functions
RGB: primaries are monochromatic, energies are 645.2nm, 526.3nm, 444.4nm. Color matching functions have negative parts -> some colors can be matched only subtractively.
CIEXYZ: Color matching functions are positive everywhere, but primaries are imaginary (i.e., not visible colors).
Color spaces

• Linear color spaces describe colors as linear combinations of primaries
• Choice of primaries = choice of color matching functions = choice of color space
• Color matching functions, hence color descriptions, are all within linear transformations

• RGB: primaries are monochromatic, energies are 645.2nm, 526.3nm, 444.4nm. Color matching functions have negative parts -> some colors can be matched only subtractively
• CIE XYZ: Color matching functions are positive everywhere, but primaries are imaginary (i.e., not visible colors).
Color Spaces

There are many different color spaces, with each describing a color using three numbers:

1. RGB
2. XYZ
3. CIExyz
4. LMS
5. CMY
6. YIQ (NTSC)
7. YUV (PAL)
8. YCbCr
9. SUV
10. CIELAB

In general a color represented in one color space (say XYZ) can be converted and represented in a second color space (say RGB)
Example: RGB to YIQ

\[
\begin{bmatrix}
Y \\
I \\
Q
\end{bmatrix} = \begin{bmatrix}
0.299 & 0.587 & 0.114 \\
0.596 & -0.275 & -0.321 \\
0.212 & -0.532 & 0.311
\end{bmatrix} \begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

- Used by NTSC TV standard
- Separates Hue & Saturation (I,Q) from Luminance (Y)
**RGB Color Cube**

- Block of colors for \((r, g, b)\) in the range \((0-1)\).
- Convenient to have an upper bound on coefficient of each primary.
- In practice:
  - primaries given by monitor phosphors (phosphors are the materials on the face of the monitor screen that glow when struck by electrons)
CIELAB

- Also referred to as CIE L*a*b*
- Designed to approximate human vision
  - Nonlinear response
- Includes 100% of visible colors
- L is lightness
- A and B are color-opponent dimensions
- Nonlinear conversion to/from CIEXYZ colorspace
- Human perceptual difference between two colors is the Euclidean distance between the two 3D points in CIELAB space
CIE\text{XYZ} and CIE\text{Exyz}

- Encompasses all color sensations the average person can experience
- Standard reference
  - Many other color space definitions are based on XYZ
- Y is luminance
- Z is quasi-equal to blue stimulation
- X is a linear combination of cone response curves chosen to be nonnegative
- The plane parallel to the XZ plane and that Y lies on contains all possible chromaticities at that luminance

\[
x = \frac{X}{X + Y + Z} \\
y = \frac{Y}{X + Y + Z} \\
z = \frac{Z}{X + Y + Z} = 1 - x - y
\]

\[
X = \frac{Y}{y} \\
Z = \frac{Y}{y}(1 - x - y)
\]
CIEXYZ and CIExy

Usually draw x, y, where
\[ x = \frac{X}{X+Y+Z} \] and
\[ y = \frac{Y}{X+Y+Z} \]

\[ z = 1 - x - y \]
CIExyY (Chromaticity Space)
Color Specification: Chromaticity

- Chromaticity coordinates
  - \((x, y, z)\)
    where \(x + y + z = 1\)
  - Usually specified by \((x, y)\)
    where \(z = 1 - x - y\)
Chromaticities

- Set of chromaticities
  - Red
  - Green
  - Blue
  - White (point)
## Standard Illuminants

<table>
<thead>
<tr>
<th>Name</th>
<th>CIE 1931 2°</th>
<th>CIE 1964 10°</th>
<th>CCT (K)</th>
<th>Hue</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_2$</td>
<td>$y_2$</td>
<td>$x_{10}$</td>
<td>$y_{10}$</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.44757</td>
<td>0.40745</td>
<td>0.45117</td>
<td>0.40594</td>
<td>2856</td>
</tr>
<tr>
<td>B</td>
<td>0.34842</td>
<td>0.35161</td>
<td>0.34980</td>
<td>0.35270</td>
<td>4874</td>
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<tr>
<td>C</td>
<td>0.31006</td>
<td>0.31616</td>
<td>0.31039</td>
<td>0.31905</td>
<td>6774</td>
</tr>
<tr>
<td>D50</td>
<td>0.34567</td>
<td>0.35850</td>
<td>0.34773</td>
<td>0.35952</td>
<td>5003</td>
</tr>
<tr>
<td>D55</td>
<td>0.33242</td>
<td>0.34743</td>
<td>0.33411</td>
<td>0.34877</td>
<td>5503</td>
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<tr>
<td>D65</td>
<td>0.31271</td>
<td>0.32902</td>
<td>0.31382</td>
<td>0.33100</td>
<td>6504</td>
</tr>
<tr>
<td>D75</td>
<td>0.29902</td>
<td>0.31485</td>
<td>0.29968</td>
<td>0.31740</td>
<td>7504</td>
</tr>
<tr>
<td>E</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>5454</td>
</tr>
<tr>
<td>F1</td>
<td>0.31310</td>
<td>0.33727</td>
<td>0.31811</td>
<td>0.33559</td>
<td>6430</td>
</tr>
<tr>
<td>F2</td>
<td>0.37208</td>
<td>0.37529</td>
<td>0.37925</td>
<td>0.36733</td>
<td>4230</td>
</tr>
<tr>
<td>F3</td>
<td>0.40910</td>
<td>0.39430</td>
<td>0.41761</td>
<td>0.38324</td>
<td>3450</td>
</tr>
<tr>
<td>F4</td>
<td>0.44018</td>
<td>0.40329</td>
<td>0.44920</td>
<td>0.39074</td>
<td>2940</td>
</tr>
<tr>
<td>F5</td>
<td>0.31379</td>
<td>0.34531</td>
<td>0.31975</td>
<td>0.34246</td>
<td>6350</td>
</tr>
<tr>
<td>F6</td>
<td>0.37790</td>
<td>0.38835</td>
<td>0.38660</td>
<td>0.37847</td>
<td>4150</td>
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<tr>
<td>F7</td>
<td>0.31292</td>
<td>0.32933</td>
<td>0.31569</td>
<td>0.32960</td>
<td>6500</td>
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<tr>
<td>F8</td>
<td>0.34588</td>
<td>0.35875</td>
<td>0.34902</td>
<td>0.35939</td>
<td>5000</td>
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<tr>
<td>F9</td>
<td>0.37417</td>
<td>0.37281</td>
<td>0.37829</td>
<td>0.37045</td>
<td>4150</td>
</tr>
<tr>
<td>F10</td>
<td>0.34609</td>
<td>0.35986</td>
<td>0.35090</td>
<td>0.35444</td>
<td>5000</td>
</tr>
<tr>
<td>F11</td>
<td>0.38052</td>
<td>0.37713</td>
<td>0.38541</td>
<td>0.37123</td>
<td>4000</td>
</tr>
<tr>
<td>F12</td>
<td>0.43695</td>
<td>0.40441</td>
<td>0.44256</td>
<td>0.39717</td>
<td>3000</td>
</tr>
</tbody>
</table>

Hue of each white point, calculated with luminance $Y = 0.54$
Chromaticity Diagrams

Rec. 709 and sRGB
35.9% of visible colors

Adobe RGB
52.1% of visible colors
Chromaticity Diagrams

Rec. 709 and sRGB
35.9% of visible colors

Wide gamut RGB
77.6% of visible colors
Chromaticity Diagrams

Rec. 709 and sRGB
35.9% of visible colors

ProPhoto RGB
90% of visible colors
Academy Color Encoding Specification (ACES)

<table>
<thead>
<tr>
<th>Color</th>
<th>CIE x</th>
<th>CIE y</th>
<th>CIE z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>0.73470</td>
<td>0.26530</td>
<td>0.00000</td>
</tr>
<tr>
<td>Green</td>
<td>0.00000</td>
<td>1.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Blue</td>
<td>0.00010</td>
<td>-0.07700</td>
<td>1.07690</td>
</tr>
<tr>
<td>White</td>
<td>0.32168</td>
<td>0.33767</td>
<td>0.34065</td>
</tr>
</tbody>
</table>

ACES is approximately D60, covering 100% of visible colors.
Incorrect Image Conversion

Same pixel values stored in files, but with different sets of chromaticities
Chromatic Adaptation

- Estimating the appearance of a sample under a different illuminant
  - Convert between different white points
- LMS color space
  - Response of the three types of cones in the human eye
    - Long, medium, and short wavelengths
- XYZ to LMS
  - Bradford transformation matrix
- Chromatic adaptation
  - Adaptation matrix

\[
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
0.8951 & 0.2664 & -0.1614 \\
-0.7502 & 1.7135 & 0.0367 \\
0.0389 & -0.0685 & 1.0296
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

\[
D = D_1^{-1}D_2 = \begin{bmatrix}
L_2/L_1 & 0 & 0 \\
0 & M_2/M_1 & 0 \\
0 & 0 & S_2/S_1
\end{bmatrix}
\]
Application: Color Transfer

- RGB to XYZ with white point of standard illuminant E (use chromatic adaptation)
- XYZ to Lab
- Map source pixels such that the L*a*b* mean and standard deviations match those of the target image
Nonlinear Encoding

• All of these standards use nonlinear encoding (gamma encoding)
  – Video
    • Recommendation ITU-R BT.601 (standard-definition television (SDTV))
    • SMPTE standard 240M (precursor to Rec. 709)
    • Recommendation ITU-R BT.709 (high-definition television (HDTV))
  – Image
    • sRGB
    • Adobe RGB
    • Wide gamut RGB (or Adobe Wide Gamut RGB)
    • ProPhoto RGB (or reference output medium metric (ROMM) RGB)

• Must convert to linear colorspace first for most color processing
Nonlinear Encoding and Conversion to Linear

• Typical CRT monitors have a transfer function of gamma = 2.2
• Image and video standards were designed to be directly displayed on CRTs
  – Pixel values are encoded to approximate gamma = 2.2
• Nonlinear to linear (floating-point) using a lookup table
• Linear to nonlinear calculation
Nonlinear R'G'B' Color Space and Linear RGB Color Space

- Example: sRGB

Slope of sRGB nonlinear in log-log space

![Graph showing the slope of sRGB nonlinear in log-log space compared to RGB linear.](image)
Luminance Y and Luma Y'

• Luminance is different than Luma
• Luminance is calculated from linear RGB
  – Y coordinate of XYZ
• Luma is calculated from nonlinear R'G'B'
• Example: sRGB
  \[ Y = 0.21263903 \times R + 0.71516871 \times G + 0.072192319 \times B \]
  \[ Y' = 0.21263903 \times R' + 0.71516871 \times G' + 0.072192319 \times B' \]
Next Lecture

- Human visual system