ECE 259A: Problem Set #1

0. Send an e-mail to avardy@ucsd.edu, stating your name, your general research interests, your research advisor (if you have one), and what brings you to the course on algebraic coding theory.

1. An erasure is an error whose location is known. Prove that a code $C$ over $\mathbb{F}_q$ with minimum distance $d$ can correct $\tau$ errors and $\sigma$ erasures, provided $d \geq 2\tau + \sigma + 1$.

2. Find all the binary linear MDS codes. Prove your answer.

3. Consider the binary linear code defined by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) Find a generator matrix for this code in systematic form.
(b) Find a parity check matrix for this code.
(c) What is the minimum distance of this code?

4. Let $C$ be an $(n, k, 2t + 1)$ binary linear code with parity check matrix $H$. Let $C'$ be a code of length $n+1$ obtained by appending to all the codewords $x = (x_1, x_2, \ldots, x_n) \in C$ an overall parity-check bit equal to $x_1 + x_2 + \cdots + x_n$.

(a) Find the parameters of $C'$.
(b) Express the parity-check matrix for $C'$ in terms of $H$.

5. Show that in a binary linear code, either all the codewords have even weight or exactly half have even weight and half have odd weight. Further show that either all the codewords contain zero in a given position, or half have zero and half have one.

6. A pointer to the table of bounds on $A_2(n, d)$, the maximum number of codewords in any binary code of length $n$ and minimum distance $d$, can be found on the class website. However, this table specifies the bounds on $A_2(n, d)$ only for even $d$.

(a) Prove that for odd $d$, we have $A_2(n, d) = A_2(n+1, d+1)$.
(b) Optional for extra credit: prove that $A_2(n, d)$ itself must be even.

7. Let $C^\perp$ be the dual of an $(n, k, d)$ binary linear code $C$. Let $H$ and $G$ be the parity-check and generator matrices for $C$, respectively. Prove that:

(a) $G$ is a parity-check matrix for $C^\perp$.
(b) $H$ is a generator matrix for $C^\perp$.
(c) $\dim C^\perp = n - k$.
(d) $(C^\perp)^\perp = C$.

8. Let $H$ be a systematic parity check matrix of an $(n, k, 2t+1)$ code $C$. Assume that $y$ was received from the channel and the syndrome $s = Hy^t$ has weight $\leq t$. Show that the only possible error pattern of weight $\leq t$ is $e = (0|s)$, where $0$ denotes the all-zero vector of length $k$. 
9. Consider the code whose parity check matrix is given by:

\[ H = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \]

(a) Write-down a standard array for the code.
(b) Decode the vector 00111. Is the answer unique? If not, find all possible answers.