CSE 200 – Fall 2021
Homework 2
Due Monday, November 1, 11:59pm

Question 1: Proving NP Completeness

An undirected graph G is k-colorable if there is a way to color its vertices with k colors, such that two adjacent nodes get different colors.
Formally: G=(V,E) is k-colorable if there exists a map $c: G \rightarrow \{1, \ldots, k\}$ such that for all edges $(u, v) \in E$ it holds that $c(u) \neq c(v)$.
Let k-COL={G: G is k-colorable} be the language of k-colorable graphs.

(a) Prove that 3-COL is NP complete.
(b) Prove that 2-COL is in P.

Question 2: Collapses of the polynomial hierarchy

Prove that if for some $i \geq 1$ it holds that $\Sigma_i = \Pi_i$ then $PH = \Sigma_i = \Pi_i$, that is the polynomial hierarchy collapses to the i-th level.

Question 3: Co-NP Completeness

Recall that:

1. A language L is in coNP if its complement L^c is in NP.
2. A language L is coNP-hard if for any language L' in coNP, there is a poly-time reduction from L' to L.
3. A language L is coNP-complete if it is both in coNP and coNP-hard.

Prove that L is coNP-complete iff its complement L^c is NP-complete.

Question 4: Designing algorithms in logspace

Consider the language $SUMEQUAL = \{x\#y\#z : x, y, z \in \{0,1\}^*, x + y = z\}$. Here, we consider x,y,z as integers represented in binary, and # is a special character that separates them. Prove that $SUMEQUAL$ is computable in logarithmic space (that is, $SUMEQUAL \in L$).

Question 5:

Recall the NL-complete language CONN:

CONN={(G,s,t): G is a directed graph, s,t are nodes in G, there is a path in G from s to t}.

Assume G has n nodes. There are two families of algorithms to solve CONN:

1. BFS/DFS based algorithms. These use $O(n)$ space and polynomial time (concretely $O(|E|) = O(n^2)$ time).
2. Savitch's algorithm (Note 5, Thm 5.1) which uses $O(log^2 n)$ space.

(a) How much time does Savitch's algorithm need? Does it run in polynomial time? Why or why not? Hint: express the asymptotic time complexity of Savitch.

(b) If you restrict your algorithm to run in poly-time, what is the minimal amount of space you can achieve? Can you beat the linear space used by BFS/DFS?

There is no "textbook solution" for this question. Instead, I want to see your best effort and creative ideas.