Today's learning goals  Sipser Ch 1.1

• Use and design a finite automaton via its
  - formal definition
  - state diagram
• Identify the strings and languages accepted by a given finite automaton
• Design a finite automaton which accepts a given language
Review: Finite automaton

• Input: finite string over a fixed alphabet
• Output: "accept" or "reject"

Computation: sequence of states traversed by the machine

Language of the machine is the set of strings it accepts
A **finite automaton** is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\) where

1. \(Q\) is a finite set called the states
2. \(\Sigma\) is a finite set called the alphabet
3. \(\delta : Q \times \Sigma \rightarrow Q\) is the transition function
4. \(q_0 \in Q\) is the start state
5. \(F \subseteq Q\) is the set of accept states.

No circles and arrows, same information!
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Can there be more than one start state in a finite automaton?

A. Yes, because of line 4.
B. No, because of line 4.
C. I don't know
Finite automaton

A finite automaton is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\) where

1. \(Q\) is a finite set called the states
2. \(\Sigma\) is a finite set called the alphabet
3. \(\delta : Q \times \Sigma \to Q\) is the transition function
4. \(q_0 \in Q\) is the start state
5. \(F \subseteq Q\) is the set of accept states

Can there be zero many accept states?

A. Yes, in which case the language is empty.
B. Yes, in which case the language is all strings.
C. No, because of line 5.
D. I don't know.
Finite automaton

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Can one state have two different transitions labelled with the same symbol going out of it?

A. Yes, because of 2.
B. Yes, because of 3.
C. No, because of 2.
D. No, because of 3.
E. I don't know.
A finite automaton is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\) where

1. \(Q\) is a finite set called the states
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How many outgoing arrows from each state?
A. May be different number at each state.
B. Must be 2.
C. Must be \(|Q|\).
D. Must be \(|\Sigma|\).
E. I don't know.
An example

\[(\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_4\})\]

What's the best description of the language recognized by this DFA?

A. Start with b and ends with a or b
B. Starts with a and ends with a or b
C. a's followed by b's
D. More than one of the above
E. I don't know.
An example

\[ \{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_4\} \]

This DFA recognizes
the language of all strings
of the form “a's followed by b's”
i.e. \[ \{ a^n b^k | n, k \geq 1 \} \]
Another example

What is the best description of language recognized by this automaton?

A. \{ a^n b^k \mid n, k \geq 1 \}
B. \{ a^n b^k \mid n \geq 1, b \geq 0 \}
C. \{ awb \mid w \text{ in } \{a, b\}^* \}
D. \{ aw \mid w \text{ in } \{a, b\}^* \}
E. I don't know
A language is regular if there is some finite automaton that recognizes exactly it.
Vocabulary review  From CSE20, etc. See Chapter 0

• \( \{ a,b,c,d,e \} \)  
  The set whose elements are a,b,c,d,e

• \( \{ a,b \}^* \)  
  The set of finite strings over a,b
    • Includes empty string
    • Includes a, aa, aaa
    • Includes b, bb, bbb
    • Includes ab, ababab, aaaaaaabbb
    • Does not include infinite sequences of a's and b's
    • Has infinitely many different elements

• \( | \text{ababab} | = 6 \)  
  The length of the string ababab is 6

• \( | \{ a,b,c,d,e \} | = 5 \)  
  The size of the set \( \{a,b,c,d,e\} \) is 5
Specifying an automaton

( \{q_1,q_2,q_3\}, \{a,b\}, \delta, q_1, ? )

What's the best representation of \( \delta \) for this DFA?

A. \( q_1 \rightarrow b, \ q_1 \rightarrow a, \ q_2 \rightarrow a, \ q_2 \rightarrow b, \ q_3 \rightarrow a,b. \)
B. \( \{ (q_1,b,q_1),(q_1,a,q_2),(q_2,a,q_2),(q_2,b,q_3), (q_3,a,q_3),(q_3,b,q_3) \} \)
C. \( \delta(b) = \text{same}, \ \delta(a) = \text{change} \)
D. No description other than the state diagram (circles & arrows) is possible.
E. I don't know.
Specifying an automaton

\( ( \{q1,q2,q3\}, \{a,b\}, \delta, q1, ? ) \)

What state(s) should be in \( F \) so that the language of this machine is \{ \( w \mid \text{ab is a substring of } w \} \)?

A. \{q2\}
B. \{q3\}
C. \{q1,q2\}
D. \{q1,q3\}
E. I don’t know.
Specifying an automaton

\( ( \{q1,q2,q3\}, \{a,b\}, \delta, q1, ? ) \)

What state(s) should be in \( F \) so that the language of this machine is \( \{ w \mid b's \text{ never occur after a's in } w \} \)?

A. \( \{q2\} \)
B. \( \{q3\} \)
C. \( \{q1,q2\} \)
D. \( \{q1,q3\} \)
E. I don't know.
• **Alphabet**: nonempty finite set of **symbols**
• **String** over an alphabet: finite sequence of symbols
• **Language** over an alphabet: some set of strings

• **DFA** over an alphabet: deterministic finite automaton
  • Input: finite string over a fixed alphabet
  • Output: "accept" or "reject"
  • \( L(M) = \{w \mid M \text{ accepts } w\} \)
• **Regular language**
  language that is \( L(M) \) for some DFA \( M \)
Building DFA

Typical questions
e.g. Homework, exams

Define a DFA which recognizes the given language $L$.

or

Prove that the (given) language $L$ is regular.
Building DFA

Example
Define a DFA which recognizes

\( \{ w | w \text{ has at least 2 } a's \} \)
Building DFA

Example

Define a DFA which recognizes

\[ \{ w | w \text{ has at most 2 } a's \} \]
Building DFA

Remember

States are our only (computer) memory.

Design and pick states with specific roles / tasks in mind.

“Have not seen any of desired pattern yet”

“Trap state”
Is there an infinite regular language?

A. No: all regular languages have to be finite.
B. Yes: all regular sets are infinite.
C. Yes: all infinite sets of strings over an alphabet are regular.
D. Yes: some infinite sets of strings over each alphabet are regular and some are not.
E. I don't know.
Regular languages: general facts

Is every finite language regular?

A. No: some finite languages are regular, and some are not.
B. No: there are no finite regular languages.
C. Yes: every finite language is regular.
D. I don't know.
Regular languages: general facts

True/False: each DFA recognizes a unique language. I.e. if two DFA are different (different number of states or different initial state, or different transition function, etc.) then they recognize different languages.

A. True can you prove it?
B. False can you prove it?
C. I don't know.
Next time

- Homework 1 **due this Wednesday!**
  - Set up course tools: Gradescope, Piazza, JFLAP
  - Complete the assignment and submit by 11:59pm

- Closure Properties of Regular Languages
  - What languages are regular?
  - Can regular languages be combined together?
  - Take a look at Homework 2