CSE 291-E: Applied Cryptography

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Fall 2020 Lecture 4

Legal Notice

The Zoom session for this class will be recorded and made available asynchronously on Canvas to registered students.

Announcements

1. HW 1 is due today! Turn it in now if you haven't yet!
2. HW 2 is out, due before class in 1 week, October 20.

Last time: Block ciphers

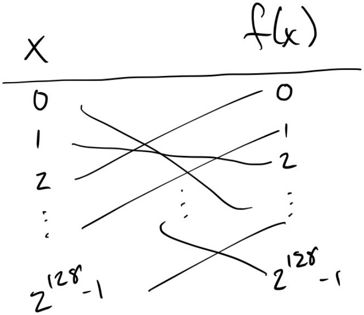
This time: Pseudorandom functions and chosen plaintext attacks

Pseudo-random functions (PRFs)

Deterministic algorithm F:

- $k \in K$, $x \in X$, $y \in Y$
- $F_k(x) = y$
- Should be computationally indistinguishable from a truly random function

Truly Random Function



In contrast to the (pseudo)random permutations from last lecture, this function is not required to be one-to-one.

Distinguishing experiment for PRFs

D $\qquad\qquad\qquad$ O

1. Chooses $f = \begin{cases} \text{random fn} \\ \text{PRF } F_k \quad k \leftarrow_R \{0,1\}^n \end{cases}$

2. query x_1

$f(x_1)$

x_2

$f(x_2)$

$\vdots$

bit $\in \{0,1\}$

Definition

F_k is a secure PRF if $\forall$ efficient D

$$|\Pr[D(F_k) = 1] - \Pr[D(\text{random fn}) = 1]| \quad \text{negligible}$$

Is a PRP indistinguishable from a PRF?

Consider a distingushing experiment between functions and permutations.

Observation: The only way to distinguish between a permutation and a non-permutation function is to find a collision, inputs so that $f(x_1) = f(x_2)$.

Let $|X| = N$. By the birthday bound, the adversary will observe a collision in outputs in a PRF with constant probability after $\sqrt{N}$ inputs.

Theorem

An adversary that makes Q queries can distinguish a random permutation from a random function with probability at most $Q^2/2N$.

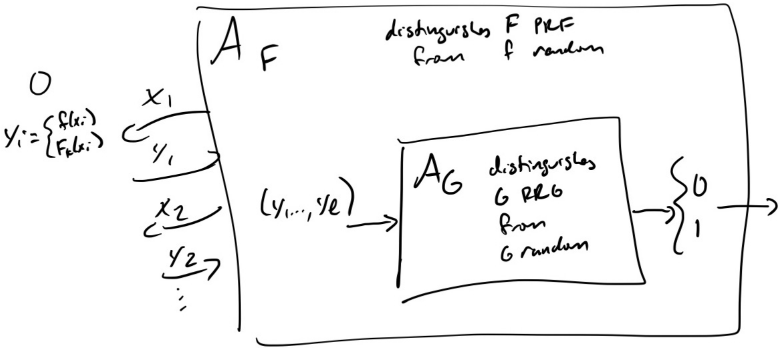
Constructing PRGs from PRFs

Theorem

Let $x_1, \ldots, x_\ell$ be fixed, distinct elements, and F be a PRF.
$G(k) = (F_k(x_1), F_k(x_2), \ldots, F_k(x_\ell))$ is a secure PRG.

Proof.

Assume for contradiction that adversary A_G can distinguish this PRG from random. Can construct a distinguisher A_F for the PRF.



Counter Mode

In the previous construction $x_1, \ldots, x_\ell$ just need to be distinct elements. So just choose r and let $x_1 = r, x_2 = r + 1, \ldots$.

stream: $F_k(r), F_k(r+1), F_k(r+2) \ldots$

Then we can use this as a stream cipher to encrypt:

$$\mathrm{Enc}_k(m) = (r, F_k(r) \oplus m[0], F_k(r+1) \oplus m[1], \ldots, F_k(r+\ell) \oplus m[\ell])$$

$$\mathrm{Dec}_k(c) = (F_k(r) \oplus c[0], F_k(r+1) \oplus c[1], \ldots, F_k(r+\ell) \oplus c[\ell])$$

This is semantically secure.

Attack models we've seen so far:

Ciphertext-only attack

- Most restrictive attack model

Known plaintext attack

- Historical example: WWII Enigma-encoded messages from Germans ending in "Heil Hitler"
- Modern example: Observing ciphertext from someone visiting the main page of Wikipedia over HTTPS.

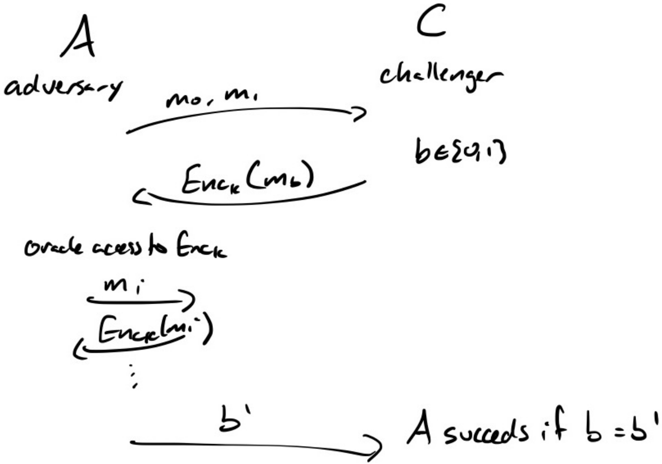
Both of these are covered by semantic security.

New attack model:

Chosen plaintext attack

- Historical example: British military would place mines in particular locations hoping Germans would send encrypted messages about that location.
- Modern example: Attacker-controlled Javascript on a web page causes victim web client to make a HTTPS connection.

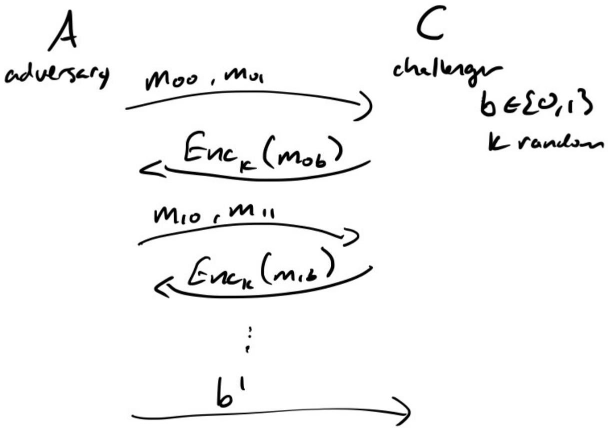
Chosen plaintext attack



Definition

Enc is CPA-secure if $\forall$ efficient A, $\Pr[A \text{ succeeds}] \leq 1/2 + \epsilon$ for ϵ negligible

Another definition of chosen plaintext attack



Definition

Enc is CPA-secure if

$|\Pr[\text{A outputs } 1 \mid b = 1] - \Pr[\text{A outputs } 1 \mid b = 0]|$ negligible

Theorem

No deterministic cipher can be CPA-secure.

$A \xrightarrow{m_0, m_1} C$

$\xleftarrow{c}$

$\xrightarrow{m_0}$

$\xleftarrow{c'}$

$\xrightarrow{m_1}$

$\xleftarrow{c''}$

Theorem

No deterministic cipher can be CPA-secure.

Proof.

Adversary queries (m_0, m_1) then (m_0, m_0). $\square$

Using a PRF for CPA-secure encryption

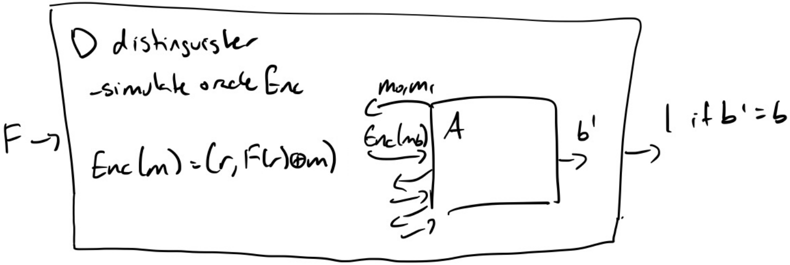
- Generate k at random.
- Encryption:
  1. Generate r uniformly at random.
  2. $\mathrm{Enc}_k(m) = (r, F_k(r) \oplus m)$
- Decryption:
  1. Parse $c = (r, s)$
  2. $\mathrm{Dec}_k(c) = F_k(r) \oplus s$.

Theorem

The $\mathrm{Enc}_k(m) = (r, F_k(r) \oplus m)$ *construction on the previous slide is CPA-secure.*

Proof.

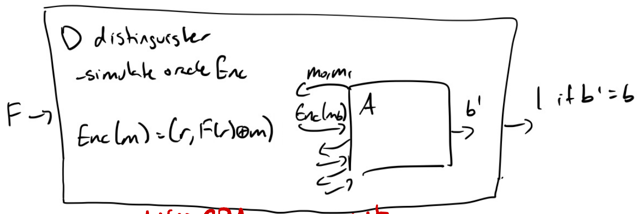
By contradiction. Assume adversary A can win CPA-security game #1 with non-negligible advantage, construct a PRF distinguisher.



□

Proof.

$|\Pr[A \mid F_k] - \Pr[A \mid f]| > \text{negl}.$



Assume A distinguishes ~~pseudorandom~~ wins CPA game against F with advantage $d >$ negl.

1. If F pseudorandom, Pr[A succeeds] $= 1/2 + d$
2. If F is a truly random function f: A makes Q oracle queries.
   - If nonce r_c used in challenge is repeated, A learns value of $f(r_c)$ and succeeds with probability 1.

$$\Pr[r_c \text{ repeated across oracle queries}] \leq Q/2^n$$

   - If r_c not used in challenge, no information: $\Pr[\text{success}] = 1/2$

$$\Pr[A \text{ succeeds}] \leq 1/2 + Q/2^n$$

$$|\Pr[D \mid \cancel{f}\, F_k] - \Pr[D \mid \cancel{F_k}\, f]| = |1/2 + d - (1/2 + Q/2^n)| = d - Q/2^n > \text{negl}.$$

Using stream ciphers in a CPA-secure way

Augment stream cipher with an *initialization vector* or IV.

- $\mathsf{Enc}_k(m) = (IV, G(k, IV) \oplus m)$

For this to be secure, $G(k, IV)$ needs to be pseudorandom when IV is known.

Insecure if IV is ever reused.

WEP insecurity. WEP is broken in multiple ways: it uses a 24-bit IV, which repeats with 50% probability after 5,000 packets.

Extension: Randomized Counter Mode

If we use a block cipher in counter mode with a randomized starting value r, this is CPA-secure.

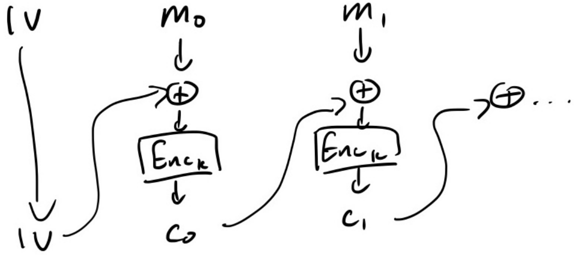
$$\mathrm{Enc}_k(m) = (r, F_k(r) \oplus m[0], F_k(r+1) \oplus m[1], \ldots, F_k(r+\ell) \oplus m[\ell])$$

$$\mathrm{Dec}_k(c) = (F_k(r) \oplus c[0], F_k(r+1) \oplus c[1], \ldots, F_k(r+\ell) \oplus c[\ell])$$

The value r is the IV.

This is an ok choice of mode of operation for AES.

Cipher block chaining (CBC) mode



1. IV has same length as block length.
2. $c_i = \text{Enc}_k(c_{i-1} \oplus m_i)$
3. Output $(IV, c_0, c_1, c_2, \ldots)$.

IV should be random.

CBC mode is CPA-secure, but suffers from implementation vulnerabilities that you get to break in HW 3.

- HW 2 is due before class in 1 week, October 20.