CSE 20, Fall 2020 - Homework 4

Due: Monday 11/09 at 11 am PDT

Instructions

Upload a single file to Gradescope for each group. All group members' names and PIDs should be on each page of the submission. You should select appropriate pages for each question when submitting to Gradescope. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Section 1.5 Table 1 (p. 60); Section 2.1, Definitions 1-3 (pp. 116-119), Definitions 6-8 (pp. 121-123); Section 2.2 Definitions 1-5 (pp. 127-129) and Table 1 (p. 130), Section 5.3 Definition of Structural Induction

Key Concepts Proof Strategies, Set Definitions and Induction

Problem 1 (20 points)

Consider Apple's newest product iPhone 14 (some point in the future). It has 4 types of memory defined as set $M = \{32G, 64G, 128G, 256G\}$ and it has 5 types of color defined as set $C = \{black, white, gray, green, orange\}$.

Customers are allowed to choose their favorite combination of memory type and color when purchasing the product. The domain of an iPhone 14 is defined as
$D = M \times C = \{(m, c) \mid m \in M,\ c \in C\}$.
All colors with the same memory type have the same price. For different memory types, the iPhone 14 are at different prices defined by the function $Price(m)$ with units in dollars:

$Price(32G) = 200$

$Price(64G) = 250$

$Price(128G) = 300$

$Price(256G) = 400$

Consider the following questions:

(a) Consider Alice wants to buy an iPhone 14. Write the following statement in first-order logic (in predicates with quantifiers), then prove or disprove the statement. (Hint: Prove the universal quantifier is true, use exhaustion or universal generalization; disprove for all quantifier statements, use counterexample) Please identify your proof strategy.

"If Alice has more than 400 dollars, then she can buy any model of iPhone 14"

(b) Prove or disprove that all green iPhone 14's are more expensive than 380 dollars. Write the statement in first-order logic (in predicates with quantifiers), then prove or disprove the statement. Please identify your proof strategy.

"All green iPhone 14's are more expensive than 380 dollars"

(c) Consider the set $A = \{(32G, black), (32G, white), (64G, orange), (128G, orange)\}$ Prove or disprove the following statement. Please identify your proof strategy.

$$\forall x,\ x \in A \rightarrow x \in D$$

Problem 2 (20 points)

Consider the Fibonnaci sequence, which is a number sequence starting with 0 and 1, then each number is the sum of the two preceding ones. Please identify your proof strategy.

(a) Prove or disprove the following claim.

"There are 3 consecutive odd Fibonnaci numbers"

(b) Prove or disprove the following claim.

"There are 3 consecutive prime Fibonnaci numbers"

Problem 3 (20 points)

Consider four sets $A = \{1, 3, 5, 6, 7\}$, $B = \{0, 1, 3, 7, 9\}$, C , and D answer the following questions:

(a) Please fill-in the blank: $A \cup B = \{ \qquad \}$

(b) Please fill-in the blank: $A - B = \{ \qquad \}$

(c) Write the power set $P(A \cap B) = \{ \qquad \}$

(d) Prove or disprove the following claim

$$A - B = B - A$$

(a) Prove or disprove the following claim. For any sets C and D:

$$P(C) \subseteq P(D) \Leftrightarrow C \subseteq D$$

Problem 4 (20 points)

Let us recall the definition for the set of linked lists L defined in class:

Basis Step: $[] \in L$

Recursive Step: If $l \in L$, and $n \in N$, then $(n, l) \in L$

We now define 3 functions on this linked list as:

i) $nodes : L \to P(N)$

Basis Step: $nodes([]) = \varnothing$

Recursive Step: If $l \in L,\ n \in N$, then $nodes((n, l)) = \{n\} \cup nodes(l)$

ii) $sum : L \to N$

Basis Step: $sum([]) = 0$

Recursive Step: If $l \in L,\ n \in N$, then $sum((n,l)) = n + sum(l)$

iii) $length: L \to N$

Basis Step: $length([]) = 0$

Recursive Step: If $l \in L,\ n \in N$, then $length((n,l)) = 1 + length(l)$

a) Consider the proposition given as: $\forall l \in L,\ \exists c \in N,\ \forall x \in N(x \leq c \to x \in nodes(l))$. Disprove the statement by proving the negation.
b) Consider the proposition given as: $\forall l \in L,\ \exists c \in N,\ \forall x \in N(x \in nodes(l) \to x \geq c)$. Prove the statement by applying structural induction on *L*.
c) Consider the proposition given as: $\forall l \in L,\ (sum(l) = length(l))$. Disprove the statement by providing a counter-example.

Problem 5 (20 points)

So far in this class, we have used recursion to define sets and functions. However, we have never offered a formal proof justifying our construction. We will now attempt to use structural induction to prove the correctness of the following functions. For each of the functions defined below, define the appropriate basis step and choose the correct recursive/inductive step to prove the correctness of the function:

a) Let us prove the correctness of the design for the set of all positive odd multiples of 4 (i.e the set {4,12,20,28…..}). To this end, we recursively define our function $multiple: N^{\neq 0} \to N$, which takes as an input an integer *x*, and computes the x^{th} odd multiple of 4 as:
Basis Step: $multiple(1) = 4$
Recursive Step: $multiple(x+1) = multiple(x) + 8$
Prove that $\forall x \in N^{\neq 0}, multiple(x) = 4(2x-1)$

b) Now, let's recall the RMSE function that we had encountered in Homework 1.
$d_{rmse,n}: ((R \times R \times ....R), (R \times R \times ......R)) \to R$
Basis Step: $d_{rmse,1}(x,y) = |x_1 - y_1|$
Recursive Step: $d_{rmse,n+1}(x,y) = \sqrt{\frac{n \times d_{rmse,n}(x,y)^2 + (x_{n+1} - y_{n+1})^2}{n+1}}$
Prove that $\forall n \in N^{\neq 0},\ [d_{rmse,n}((x_1, x_2 ....., x_n), (y_1, y_2 ....., y_n)) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - y_i)^2}{n}}]$

Problem 6 - Bonus (10 points)

Let us assume that we have the same linked list L as provided in class:

Basis Step: $[] \in L$

Recursive Step: If $l \in L$, and $n \in N$, then $(n, l) \in L$

a) Define a function *least*, which computes and returns the smallest data element in the node of the linked list (i.e., the smallest element across all nodes. Eg- (4,(1,(3,[]))) has the smallest element 1).
b) Define a function *average* that computes the mean of all data elements in the linked list. In the definition of your function, you can use any of the functions *nodes, sum* or *length* defined in Problem 4.