CSE 20, Fall 2020 - Homework 3

Due: Monday 10/26 at 11 am PDT

Instructions

Upload a single file to Gradescope for each group. All group members' names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Rosen Section 1.1 Definition 5, Definition 6, Example 11, Section 1.4 Definitions 1 (p. 40) and 2 (p. 42), Table 2 (p 47), Examples 21-22 (pp. 47-48), Section 2.1 Definitions 8 and 9 (p. 123). Section 1.5 Example 1 (p. 57) and Example 4 (p. 59)

Key Concepts Conditionals, Quantifiers and Predicates

Problem 1 (20 points)

a) Convert the following compound propositions into an equivalent statement using only the operators AND ($\wedge$), OR ($\vee$) and NOT ($\neg$)

   1) $(p \leftrightarrow q) \rightarrow r$
   2) $p \rightarrow (q \rightarrow (r \rightarrow (s \rightarrow t)))$

b) Express the negations of the following statements such that all negation symbols ($\neg$) immediately precede the predicate (i.e. write the negations as: $\neg(\exists x P(x)) \equiv \forall x(\neg P(x))$). Express your answer using equivalent statement using only the operators AND ($\wedge$), OR ($\vee$) and NOT ($\neg$).

   1) $\forall x \exists y P(x,y) \vee \exists x \forall y Q(x,y)$
   2) $\forall x \exists y (P(x,y) \rightarrow Q(x,y))$

Problem 2 (20 points)

In this question, you'll define and work with predicates defined over the set of integers A = {-1,0,1}. This will be your domain.

Recall the definition: **Statements involving predicates and quantifiers are logically equivalent means they have the same truth value no matter which predicates (domains and functions) are substituted in.**

For full credit, your solution for each part below needs to include all of the following:

1) Precise definition of predicate(s)
2) Evaluation of left-hand-side quantified statement, with explanations referring to definition of specific predicate and logical structure of statement
3) Evaluation of right-hand-side quantified statement, with explanations referring to definition of specific predicate and logical structure of statement
4) Conclusion, with explanations

a) Using an input-output definition table, define a predicate P over A so that it can be used to prove that $\exists y \forall x P(x,y)$ is not equivalent to $\forall x \exists y P(x,y)$

b) Using an input-output definition table, define a predicate Q over A so that it can be used to prove that $\forall x \exists y Q(x,y)$ is not equivalent to $\exists x \forall y Q(x,y)$

c) Determine which of the following are true logical equivalences, and informally justify your reasoning (the predicates P and Q here are independent of the previous 2 parts of the question):

   1) $\exists x P(x) \vee \exists x Q(x) \equiv \exists x (P(x) \vee Q(x))$
   2) $\forall x P(x) \vee \forall x Q(x) \equiv \forall x (P(x) \vee Q(x))$

Problem 3 (20 points)

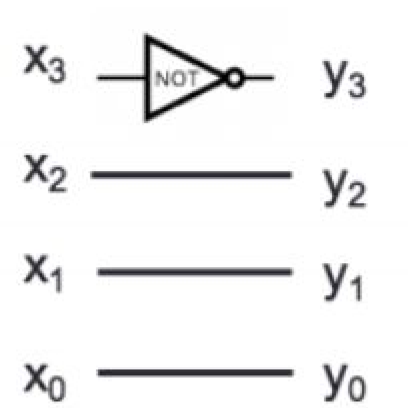
Let's say you are designing a file management system defined using the following statements:

1. If the file system is not locked, then new messages will be queued.
2. If the file system is not locked, then the system is functioning normally, and conversely.
3. If new messages are not queued, then they will be sent to the message buffer.
4. If the file system is not locked, then new messages will be sent to the message buffer.
5. New messages will not be sent to the message buffer.

a) Defining appropriate propositional variables, translate each statement into a compound proposition.
b) Determine if the system defined here is consistent.

Problem 4 (20 points)

Consider the following circuit with four inputs x_0, x_1, x_2, x_3 and four outputs y_0, y_1, y_2, y_3:



a) Does this logic circuit implement the operation of taking a number represented in width-4 sign-magnitude representation and producing the width-4 sign-magnitude representation of (−1) times this number? If yes, explain why using the definition of sign-magnitude representation and consider all possible inputs. If no, provide specific example input values, trace the circuit to compute its output for this example, and use the definition to calculate the two numbers being represented and explain why these numbers support your conclusion.
b) Does this logic circuit implement the operation of taking a number represented in width-4 sign-magnitude representation and producing the width-4 2s complement representation of (−1) times this number? If yes, explain why using the definition of 2s complement representation and consider all possible inputs. If no, provide specific example input values, trace the circuit to compute its output for this example, and use the definition to

calculate the two numbers being represented and explain why these numbers support your conclusion.

Problem 5 (20 points)

In this problem, we consider building a circuit to implement the summation of 2 width-3 signed numbers. Mathematically, this problem can be formulated as follows:
Given 2 inputs $(x_2x_1x_0)_2,\ (y_2y_1y_0)_2$, implement a circuit to calculate the summation $(z_2z_1z_0)_2$
Assume all negative numbers are using 2s complement representation.

a) Design the circuit and draw it.
b) Verify the correctness of your design. You can do this by representing $z_2,\ z_1, z_0$ as a function of $x_2,\ x_1,\ x_0,\ y_2,\ y_1,\ y_0$ and calculate the entire truth table.

Problem 6 (10 points)

In this problem, you're supposed to use only and, or, xor, and negation gates to build up circuits that satisfy specific requirements.

- Please refer to Rosen p.25 for definitions of tautology and contradiction.

a) Suppose there's only one input signal. Design a circuit made up of only one gate that represents a contradiction. If you think such a circuit doesn't exist, please provide some reasoning.
b) Suppose there's only one input signal. Design a circuit made up of only one gate that represents a tautology. If you think such a circuit doesn't exist, please provide some reasoning.