# CSE 20, Fall 2020 - Homework 2

Due: Monday 10/19 at 11 am PDT

## Instructions

Upload a single file to Gradescope for each group. All group members' names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

**Reading** Rosen Section 4.2 (p. 256) Section 1.2 (p. 20-21) Section 1.1 (p. 4-6) Section 1.3 (definition 1 and 2)

**Key Concepts** One's, Two's complement, logic gates and circuits, truth table, compound proposition

## Problem 1 (20 points)

(a) Convert the following numbers to another representation in binary. Note that they are fixed width. Numbers are written in the following form:

$$[n]_{width,\ representation}$$

Example: $[0010]_{4,\ sign-magnitude}$ is a decimal 2 in sign-magnitude of width 4.

(i) 27 = [ $]_{8,\ \text{sign-magnitude}}$

(ii) -133 = [ $]_{12,\ \text{two's complement}}$

(iii) 74 = [ $]_{8,\ \text{two's complement}}$

(iv) -35 = [ $]_{8,\ \text{two's complement}}$

(b) What are the pros and cons of using signed magnitude and two's complement?

(i) pros of signed magnitude

(ii) cons of signed magnitude

(iii) pros of two's complement

(iv) cons of two's complement

## Problem 2 (20 points)

RGBA, red green blue alpha, is a commonly used color space in computer science. Such representation can be written as a 4-tuple, (*r, g, b, a*), where *r* represents the red component, *g* the green component, *b* the blue component, and *a* the transparency component alpha. Each of the four values must be an element from the set $A = \{x \in \mathbb{N} \mid 0 \le x \le (255)_{10}\}$ .

**Definition**: A RGBA color is a nonnegative integer, *n*, that has a base 16 fixed-width 8, and can be defined as

$$n = (r_1 r_2 g_1 g_2 b_1 b_2 a_1 a_2)_{16,8}$$

And $A$ is defined above

(a) Why is each color component represented in 2 digits of hexadecimal number? Please give 1-2 sentences description.

(b) What is the green and alpha component of RGBA color $(2404399377)_{10}$ ? Hint: Convert the number to hexadecimal and use the definition above. For example, $(FF)_{16,2}$ is the red component in RGBA color $(FFFFFFFF)_{16,8}$

(c) If representing a RGBA color defined above (with hexadecimals) takes 8 bytes in the system, then how many bytes would representing a RGBA color take if the digits are in decimal? Assuming we want each component to be represented separately and each decimal digit takes 1 byte, for example $(0, 50, 70, 255)_{10}$ will be an example of RGBA color in decimal.

(d) What is $(FF0000FF)_{16,8}$ (opaque red) subtract $(EE82EE80)_{16,8}$ (50.2% transparent violet) in decimal? Write down your explanation and how you convert to decimal.

## Problem 3 (20 points)

(a) Choose the appropriate answer and fill in the blanks. Each option might be used more than once or unused at all.

DNF (disjunctive normal form) are _____ of _____(s)
CNF (conjunctive normal form) are _____ of _____(s)

(A) AND (B) OR (C) NOT (D) unknowns

(b) Consider the following truth table

| p | q | r | Output |
|---|---|---|---|
| T | T | T | T |
| T | T | F | F |
| T | F | T | F |
| T | F | F | T |
| F | T | T | F |
| F | T | F | F |
| F | F | T | T |
| F | F | F | T |

(i) Write down DNF describing the above table
(ii) Write down CNF describing the above table

## Problem 4 (20 points)

a) In class, we used the XOR gate to construct several logic circuits such as the full-adder circuit. Let us say you are now asked to design an XOR gate, using only NOT and OR gates. Write a logically equivalent compound proposition, using only $\neg$ and $\vee$ to represent the XOR operation:

$$z = p \oplus q$$

Draw a truth table to verify your answer.

b) The 3-variable XOR function is defined as:

$$z = p \oplus q \oplus r$$

Simplify the propositional logic for *z* in either the CNF or DNF form by drawing a truth table.

## Problem 5 (20 points)

For this question, we will consider the logic circuit for multiplying two two-bit binary numbers (we will not be considering the numbers to be signed for this problem). For this purpose, we define the result as:

$$(z_3z_2z_1z_0)_{2,4} = (x_1x_0)_{2,2} \times (y_1y_0)_{2,2}$$

with $(x_1x_0)_{2,2}$ and $(y_1y_0)_{2,2}$ the input numbers in binary.

a) Write the input-output definition table (truth table) for this operation: your table should have 16 rows of inputs, and 4 outputs $(z_3, z_2, z_1, z_0)$ for each operation.
b) Write a compound proposition that is logically equivalent to the expression for $z_3, z_2, z_1$ and $z_0$. *Hint:* CNF and DNF forms may be useful here.

## Problem 6 (Bonus: 10 points)

Simplification of logical propositions is often a challenging aspect of digital design. You are given the following truth table:

| *p* | *q* | *r* | Output |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Express the output as a compound proposition whose circuit uses exactly 2 gates. You are allowed to use AND $(\wedge)$, OR $(\vee)$, NOT $(\neg)$ and XOR $(\oplus)$.