

CSE 20, Fall 2020 - Midterm 2 Review

Question 1

Prove that for any real number $x > -1$, and any positive integer n , $(1+x)^n \geq 1+nx$.

Question 2

Let U be the universal set, and let A , B and C be sets. Use the definition of set equality to prove: $A - (B \cup C) = (A - B) - C$

Question 3

If $n \bmod 3 = 2$, then show that $n^2 + 2n$ is not divisible by 3.

Question 4

For each of the following relations R on the given domain A , categorize them as one of the following:

- Not an equivalence relation
- An equivalence relation with finitely many distinct equivalence classes
- An equivalence relation with infinitely many distinct equivalence classes

a) $A = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3)\}$

b) $A = \mathbb{R}$, $R = \{(x, y) \mid x^2 = y^2\}$

c) $A = Z \times Z^*$, $R = \{(a, b), (c, d) \mid ad = bc\}$, where Z^* are the set of non-zero integers

Question 5

Calculate $7^{15420} \bmod 13$.

Question 6

Prove that for three integers a, b and c , if c does not divide $a \times b$ then c does not divide a and c does not divide b .

Question 7

For $f_1, f_2 : A \rightarrow B$ and $g : B \rightarrow C$, prove that if g is injective and $g(f_1(x)) = g(f_2(x))$ for all $x \in A$, then $f_1 = f_2$.