CSE 20 Discussion
Week 6

TA

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Induction Family

- Mathematical Induction

To prove a universal quantification where the element comes from the set of integers >= b, prove two cases:

1. Prove the property is true about the number b

2. Consider an arbitrary integer n greater than or equal to b, assume (as the **induction hypothesis**) that the property holds for n, and use this and other facts to prove that the property holds for n+1.

- Strong Induction

To prove a universal quantification where the element comes from the set of integers >= b:

1. Pick j basis cases and prove the property is true about b, …, b+j

2. Consider an arbitrary integer n that is >= b, assume (as the **strong induction hypothesis** that the property holds for each of b, …, n, and use this and other facts to prove that the property holds for n+1.

- Structural Induction

To prove a universal quantification where the element comes from a recursively defined set, prove two cases:

1. Assume the element is one of those from the basis step and prove the conclusion

2. Assume the element is one of those from the recursive step, and assume that the property holds for the elements used to build it, and prove the conclusion.

Let $P(n)$ be the statement that

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

where n is an integer greater than 1.

- Suppose we want to do this in an induction manner
- What is the basis step?
- Previously we only deal with equations. What should be modified to prove an inequation?
- How to move advance to new value of n?

Prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

- What comes into your mind when you're supposed to prove that F(x) mod k =0 for all x?
- 21 seems like a big number. If we want to enumerate, will that be too difficult? How to break down 21?

Show that $\mathrm{n}, n \geq 1$ lines divide the plane into $\frac{n^2+n}{2} + 1$ regions if every two lines have exactly one common point and no three lines contain a common point

- When you’re supposed to prove something with arbitrary n, and it seems difficult, it’s always a good idea to start from small n
- Let’s draw some examples with small n