CSE 20 Discussion
Week 5

TA

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Agenda

- Proof
- Induction

Proofs

- Two types of assertion
  - “There exist something…”
  - ”For all items in a set, something holds”

- To prove that $\forall x P(x)$ is true, use exhaustion or universal generalization.
- To prove that $\forall x P(x)$ is false, use a counterexample.
- To prove that $\exists x P(x)$ is true, use a witness.
- To prove that $\exists x P(x)$ is false, write universal statement that is logically equivalent to its negation and then prove it true using universal generalization.

Prove or Disprove The Following Proposition

$$\forall x, y \in \mathbb{Z}, (IsEven\ (x \cdot y) \wedge IsEven(x + y)) \rightarrow (IsEven(x) \wedge IsEven(y))$$

- How do you understand (interpret) this?
- Which of the techniques described last page should be applied?
- We can proof by exhaustion!
- What else?

Prove or Disprove The Following Proposition

$$\forall x, y \in \mathbb{Z}, (IsEven\ (x \cdot y) \land IsEven(x + y)) \rightarrow (IsEven(x) \land IsEven(y))$$

- Proof by contraposition
- General case: we want to proof…
  - If A, then B
  - This equals to ‘if ~B, then ~A’

Proof & Induction

- Use structural induction to prove that l(xy) = l(x) + l(y), where x and y belong to the set of strings (S) over the alphabet (α)
- Recall the recursive definition of string length:
  - l(λ) = 0
  - If $w \in S$ and $x \in \alpha$, then l(wx) = l(w) + 1
- Structural induction
  - Let $P(y)$ be the statement that l(xy) = l(x) + l(y) holds for all $x \in S$
  - Basis Step: $P(\lambda)$ holds
  - Recursive Step: If P(y) holds, then P(yb) holds for all b$\in \alpha$

Something More About Induction & Proof

- Use mathematical induction to prove the following:

$$\sum_{i=1}^{n} i^2 = C_{n+1}^{2} + 2C_{n+1}^{3} = \frac{n(n+1)(2n+1)}{6}$$