CSE 20 Discussion

Week 4

For each quantified statement below, first translate to an English sentence.

Then, negate the **whole** statement and rewrite this negated statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

i. First statement:

$$\forall n \in \mathbb{N}\ \exists t \in R_4 \times R_4\ (\ d_{1,4}(t) = n\ )$$

ii. Second statement:

$$\forall t_1 \in R_4 \times R_4\ \forall t_2 \in R_4 \times R_4\ (\ \neg(t_1 = t_2) \to \neg(d_{2,4}(t_1) = d_{2,4}(t_2))\ )$$

1. Consider the predicate $F(a, b) =$ "a is a factor of b" over the domain $\mathbb{Z}^{\neq 0} \times \mathbb{Z}$. Consider the following quantified statements

(i) $\forall x \in \mathbb{Z}\ (F(1, x))$

(ii) $\forall x \in \mathbb{Z}^{\neq 0}\ (F(x, 1))$

(iii) $\exists x \in \mathbb{Z}\ (F(1, x))$

(iv) $\exists x \in \mathbb{Z}^{\neq 0}\ (F(x, 1))$

(v) $\forall x \in \mathbb{Z}^{\neq 0}\ \exists y \in \mathbb{Z}\ (F(x, y))$

(vi) $\exists x \in \mathbb{Z}^{\neq 0}\ \forall y \in \mathbb{Z}\ (F(x, y))$

(vii) $\forall y \in \mathbb{Z}\ \exists x \in \mathbb{Z}^{\neq 0}\ (F(x, y))$

(viii) $\exists y \in \mathbb{Z}\ \forall x \in \mathbb{Z}^{\neq 0}\ (F(x, y))$

(a) Select the statement whose translation is

"The number 1 is a factor of every integer."

or write NONE if none of (i)-(viii) work.

(b) Select the statement whose translation is

"Every integer has at least one nonzero factor."

or write NONE if none of (i)-(viii) work.

(c) Select the statement whose translation is

"There is an integer of which all nonzero integers are a factor."

or write NONE if none of (i)-(viii) work.

(d) For each statement (i)-(viii), determine if it is true or false.

2. Suppose $P(x)$ is a predicate over a domain D.

(a) True or False: To translate the statement "There are at least two elements in D where the predicate P evaluates to true", we could write

$$\exists x_1 \in D \, \exists x_2 \in D \, (P(x_1) \wedge P(x_2))$$

(b) True or False: To translate the statement "There are at most two elements in D where the predicate P evaluates to true", we could write

$$\forall x_1 \in D \, \forall x_2 \in D \, \forall x_3 \in D \, (\, (\, P(x_1) \wedge P(x_2) \wedge P(x_3) \,) \rightarrow (\, x_1 = x_2 \vee x_2 = x_3 \vee x_1 = x_3 \,) \,)$$

Let $W = \mathcal{P}(\{1,2,3,4,5\})$. The statement

$$\forall A \in W\ \forall B \in W\ \forall C \in W\ (A \cup B = A \cup C\ \rightarrow\ B = C)$$

is false. Which of the following choices for A, B, C could be used to give a counterexample to this claim? (Select all and only that apply.)

(a) $A = \{1,2,3\}, B = \{1,2\}, C = \{1,3\}$

(b) $A = \{\emptyset,1,2,3\}, B = \{1,2\}, C = \{1,3\}$

(c) $A = \{1,2,3\}, B = \{1,2\}, C = \{1,4\}$

(d) $A = \{1,2\}, B = \{2,3\}, C = \{1,3\}$

(e) $A = \{1,2\}, B = \{1,3\}, C = \{1,3\}$

Definition (Rosen p. 257): An integer p greater than 1 is called **prime** means the only positive factors of p are 1 and p. A positive integer that is greater than 1 and is not prime is called composite.

A formal definition of the predicate Pr over the domain $\mathbb{Z}$ which evaluates to T exactly when the input is prime is: $(x > 1) \wedge \forall a(\ (\ a > 0 \wedge F(a, x)\ ) \rightarrow (a = 1 \vee a = x)\ )$

Claim: 1 is not prime.

Claim: 4 is not prime.