# Week 3

CSE 20 Fall 2020

Propositional Logic

Logical Operators

# Propositional Logic

**Question: Are the following propositions equivalent?**

$(p \to q) \land (p \to r)$ and $p \to (q \land r)$

Yes, they are equivalent.

$$(p \to q) \land (p \to r) \equiv (\neg p \lor q) \land (\neg p \lor r) \equiv^{*} \neg p \lor (q \land r) \equiv p \to (q \land r)$$

*this is distributive property

# Propositional Logic

**Question: Are the following propositions equivalent?**

$(p \to r) \land (q \to r)$ and $(p \land q) \to r$

No, they are not equivalent. Too see this, consider the assignment

$$p = F, q = T, r = F$$

then

- $(p \to r) \land (q \to r) = (F \to F) \land (T \to F) = T \land F = F$
- $(p \land q) \to r = (F \land T) \to F = F \to F = T$

However,

$$(p \to r) \land (q \to r) \equiv (\neg p \lor r) \land (\neg q \lor r) \equiv^{*} (\neg p \land \neg q) \lor r$$

$$\equiv^{*} \neg(p \lor q) \lor r \equiv (p \lor q) \to r$$

*distributive property *de Morgan's Law

# Propositional Logic

**Question: Are the following propositions equivalent?**

$\neg p \to (q \to r)$ and $q \to (p \vee r)$

Yes, they are equivalent.

$$\neg p \to (q \to r) \equiv \neg p \to (\neg q \vee r) \equiv \neg(\neg p) \vee (\neg q \vee r) \equiv p \vee (\neg q \vee r)$$
$$\equiv^{*} (p \vee \neg q) \vee r \equiv^{*} (\neg q \vee p) \vee r \equiv^{*} \neg q \vee (p \vee r) \equiv q \to (p \vee r)$$

*associative property

*commutative

# Propositional Logic

**Simplify the following truth table:**

| $p$ | $q$ | $r$ | ? |
|---|---|---|---|
| T | T | T | F |
| T | T | F | T |
| T | F | T | F |
| T | F | F | F |
| F | T | T | T |
| F | T | F | F |
| F | F | T | F |
| F | F | F | F |

$p \wedge q \wedge \neg r$

$\neg p \wedge q \wedge r$

$(\boldsymbol{p} \wedge \boldsymbol{q} \wedge \neg \boldsymbol{r}) \vee (\neg \boldsymbol{p} \wedge \boldsymbol{q} \wedge \boldsymbol{r})$

$$(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \equiv \big(q \wedge (p \wedge \neg r)\big) \vee \big(q \wedge (\neg p \wedge r)\big) \equiv q \wedge \big((p \wedge \neg r) \vee (\neg p \wedge r)\big) \equiv q \wedge (p \oplus r)$$

# Propositional Logic

Converse of $p \to q$:

Contrapositive of $p \to q$:

Inverse of $p \to q$:

p: “Maria learns discrete mathematics”

q: “Maria will find a good job.”

Express the statement $p \to q$ as a statement in English.

“If Maria learns discrete mathematics, she will find a good job”

# Propositional Logic

Write the following statement as a proposition:

"The home team wins whenever it is raining."

p: It is raining.

q: The home team wins.

$p \rightarrow q$: If it is raining, the home team wins.

Converse: $q \rightarrow p$: If the home team wins, then it is raining.

Contrapositive: $\neg q \rightarrow \neg p$: If the home team does not win, then it is not raining.

Inverse: $\neg p \rightarrow \neg q$: If it is not raining, then the home team does not win.

Observe: The contrapositive is the same as the original

# Quantifiers

Let $P(x)$ be the statement "$x$ can speak Klingon" and let $Q(x)$ be the statement "$x$ knows C++."

Express each of these sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students in our class.

a) There is a student in our class who can speak Klingon and knows C++. $\exists x(P(x) \land Q(x))$

b) There is a student in our class who can speak Klingon but doesn't know C++. $\exists x(P(x) \land \neg Q(x))$

c) No student in our class can speak Klingon or knows C++. $\neg \exists x\big(P(x) \lor Q(x)\big) \equiv \forall x(\neg P(x) \land \neg Q(x))$

# Quantifiers

What is the truth value of

$\forall n \exists m (n^2 < m)$

$\exists n \forall \mathrm{m} (n < m^2)$

The domain is the set of integers.

$\forall n \exists m (n^2 < m)$ is true.
To see this, pick an arbitrary integer $n$.
We need to show that there exists an integer $m$ such that $n^2 < m$.
Indeed, when $m = n^2 + 1$, we have $n^2 < n^2 + 1 = m$.
Hence, for each integer n, there exists an integer m such that $n^2 < m$.

$\exists n \forall \mathrm{m} (n < m^2)$ is true.
Let $n = -1$ then for all integer $m$, we have

$$n = -1 < 0 \leq m^2.$$

Thus, there exists an integer $n$ such that for all integers $m, n < m^2$.