

CSE 20 Discussion

Week 3

TA

Yuanjun “Dastin” Huang

Notations & Definitions

- Signed Representation
 - 3 ways to represent $-n$ in 2s complement with fixed width
- Digital Circuits
 - Different gates
- Definitions
 - Proposition
 - Propositional variable
 - Compound proposition
 - Truth table
- Logical Operators
 - Conjunction (and)
 - Exclusive or (xor)
 - Disjunction (or)
 - Negation (not)
- Logical Equivalence
 - \equiv
 - Tautology
 - Contradiction
- DNF and CNF
 - Disjunctive normal form (OR of ANDs)
 - Conjunctive normal form (AND of Ors)
- How to translate natural language (e.g., English) to propositional logic

Signed Representation

- 3 Ways to represent $-n$ in 2s complement

To represent $-n$ in 2s complement with width w

- Calculate $2^{w-1}-n$, convert to binary fixed-width $w-1$, pad with leading 1
- Express $-n$ as a sum of powers of 2, where leftmost (2^{w-1}) is negative weight
- Convert n to fixed-width w binary, flip bits, add 1 (ignore overflow)

Digital Circuits

- Some Terms
 - Proposition
 - Propositional variable
 - Compound proposition
 - Truth table

Gates and Operators

- Gates

- And
- Or
- Xor
- Not

- Logical Operators

- Conjunction (and)
- Exclusive or (xor)
- Disjunction (or)
- Negation (not)

Logical Equivalence

- \equiv
- Tautology
- Contradiction
- Disjunctive normal form (OR of ANDs)
- Conjunctive normal form (AND of Ors)
- Conditional
 - If p , then q
- Biconditional
 - P if and only if q

- Example
 - Let p be the statement “Maria learns discrete mathematics” and q the statement “Maria will find a good job.” Express the statement $p \rightarrow q$ as a statement in English.

Converse, Contrapositive, Inverse

CONVERSE, CONTRAPOSITIVE, AND INVERSE We can form some new conditional statements starting with a conditional statement $p \rightarrow q$. In particular, there are three related conditional statements that occur so often that they have special names. The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$. The **contrapositive** of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$. The proposition $\neg p \rightarrow \neg q$ is called the **inverse** of $p \rightarrow q$. We will see that of these three conditional statements formed from $p \rightarrow q$, only the contrapositive always has the same truth value as $p \rightarrow q$.

EXAMPLE 9 What are the contrapositive, the converse, and the inverse of the conditional statement

“The home team wins whenever it is raining?”



Solution: Because “ q whenever p ” is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as

“If it is raining, then the home team wins.”

Consequently, the contrapositive of this conditional statement is

“If the home team does not win, then it is not raining.”

The converse is

“If the home team wins, then it is raining.”

The inverse is

“If it is not raining, then the home team does not win.”

Only the contrapositive is equivalent to the original statement. 

BICONDITIONALS We now introduce another way to combine propositions that expresses that two propositions have the same truth value.