Discussion

Week 10

Review: Relations

Definition: When A and B are sets, we say any subset of $A \times B$ is a **binary relation**.

Definition: When A is a set, we say any subset of $A \times A$ is a (binary) **relation** on A.

Definition: (*Rosen 9.1*) A relation R on a set A is called **reflexive** means $(a, a) \in R$ for every element $a \in A$.

Definition: (*Rosen 9.1*) A relation R on a set A is called **symmetric** means $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Definition: (*Rosen 9.1*) A relation R on a set A is called **transitive** means whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Definition: (*Rosen 9.5*) A relation is an **equivalence relation** if it is reflexive, symmetric, and transitive.

Definition: (*Rosen 9.5*) An **equivalence class** of an element $a \in A$ for an equivalence relation R on the set A is the set $\{s \in A | (a, s) \in R\}$. We write this as $[a]_R$.

1. Recall that in a movie recommendation system, each user's ratings of movies is represented as a n-tuple (with the positive integer n being the number of movies in the database), and each component of the n-tuple is an element of the collection $\{-1, 0, 1\}$.

   Assume there are five movies in the database, so that each user's ratings can be represented as a 5-tuple. Let R be the set of all ratings, that is, the set of all 5-tuples where each component of the 5-tuple is an element of the collection $\{-1, 0, 1\}$.

   Consider the following two equivalence relations on R:

$$H = \{(u, v) \in R \times R \mid \text{users } u \text{ and } v \text{ dislike the same number of movies}\}$$

$$A = \{(u, v) \in R \times R \mid \text{users } u \text{ and } v \text{ agree about the first movie in the database}\}$$

   Extra practice: Prove that each of the above relations are equivalence relations.

   Recall that the **equivalence class** of an element $x \in X$ for an equivalence relation $\sim$ on the set X is the set $\{s \in X | (x, s) \in \sim\}$. We write this as $[x]_\sim$.

   Additionally, a **partition** of set A is a set of non-empty disjoint subsets $A_1, A_2, ..., A_n$ such that $A_1 \cup A_2 \cup ... \cup A_n = A$.

   (a) What is the equivalence class of $[(-1, -1, -1, -1, -1)]_H$?

   (b) Give the equivalence classes $[(1, 0, 0, 0, 0)]_A$, $[(0, 0, 0, 0, 0)]_A$, and $[(-1, 0, 0, 0, 0)]_A$ in set-builder notation. How large are these equivalence classes?

   (c) Give a partition of R which has 3 elements.

2. Fill in the definition for the predicate $isPalindromic(s)$ which takes an RNA strand s and returns T if s is the same strand as its reverse and F otherwise.

			$isPalindromic : S$	$\to \{T, F\}$
Basis Step:	if b $\in B$		$isPalindromic(b)$	$= T$
	if b$_1 \in B$ and b$_2 \in B$		$isPalindromic(b_1 b_2)$	$=$
Recursive Step:	If	, then	$isPalindromic(\quad)$	$=$

(a) Trace the evaluation of $isPalindromic$(AUCUA)

(b) Trace the evaluation of $isPalindromic$(AUCGUA)

(c) Why did we need the second basis step in this definition?

3. Let $P(n)$ be the statement $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$. Using induction we will show that $\forall n \in \mathbb{Z}^+ P(n)$.

   (a) Show that P(1) is true, completing the basis step

   (b) What is the inductive hypothesis?

   (c) What do you need to prove in the inductive step?

   (d) Complete the inductive step.

4. Translate these specifications into English where F(p) is “Printer p is out of service,” B(p) is “Printer p is busy,” L(j) is “Print job j is lost,” and Q(j) is “Print job j is queued.”

   (a) $\exists p(F(p) \land B(p)) \rightarrow \exists j L(j)$

   (b) $\forall p B(p) \rightarrow \exists j Q(j)$

   (c) $\exists j(Q(j) \land L(j)) \rightarrow \exists p F(p)$

   (d) $(\forall p B(p) \land \forall j Q(j)) \rightarrow \exists j L(j)$