CSE 20
DISCRETE MATH

Fall 2020

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

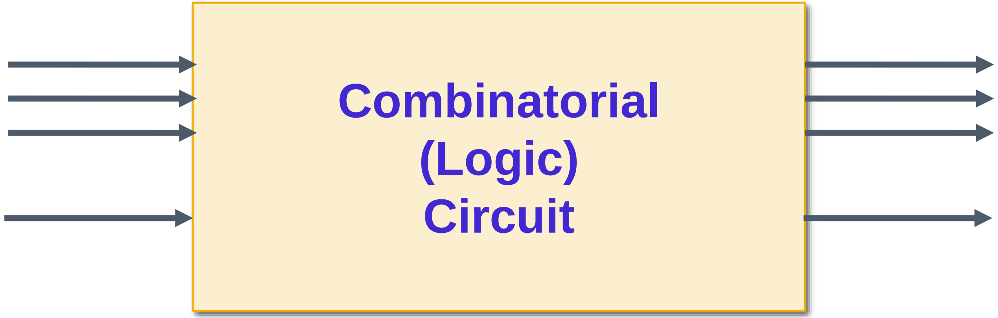
Today's learning goals

- Relate algorithms for integer operations to bitwise boolean operations
- Correctly use XOR and bit shifts
- List the truth tables and meanings for negation, conjunction, disjunction, exclusive or, implication.
- Relate boolean operations to applications in combinatorial circuits.

Arithmetic in hardware

Other models are possible

Inputs e.g. coefficients in fixed-width binary representation

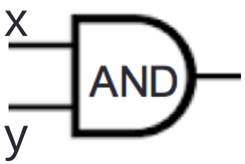


Outputs e.g. coefficients in fixed-width binary representation

Values flow left to right: possible values on a wire are 0 (low) or 1 (high)
Circuit elements: wires, gates
Gates may share input; outputs of gate can become inputs to other gates

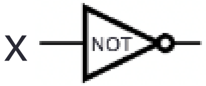
Definition tables

Input		Output
x	y	x AND y
1	1	1
1	0	0
0	1	0
0	0	0

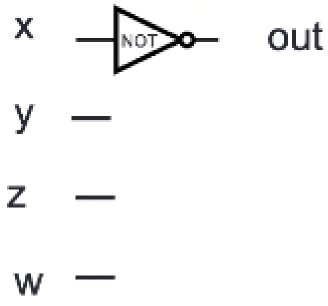


Input		Output
x	y	x XOR y
1	1	0
1	0	1
0	1	1
0	0	0

Input	Output
x	NOT x
1	0
0	1

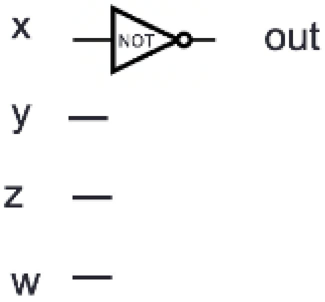


Example: logical circuit



$$\text{Output when } x = 1, y = 0, z = 0, w = 1 \text{ is } \_\_\_\_\_\_$$
$$\text{Output when } x = 1, y = 1, z = 1, w = 1 \text{ is } \_\_\_\_\_\_$$
$$\text{Output when } x = 0, y = 0, z = 0, w = 1 \text{ is } \_\_\_\_\_\_$$

Example: logical circuit



Which of the following is true about all possible input values x,y,z,w? “The output out is set to 1 exactly when

A. x is 0, and is set to 0 otherwise”
B. $(xyzw)_2$ is less than 8, and is set to 0 otherwise”
C. $(wzyx)_{2,4}$ is an even integer, and is set to 0 otherwise”
D. All of the above
E. None of the above

Circuits

Input		Output
x	y	x AND y
1	1	1
1	0	0
0	1	0
0	0	0

Input		Output
x	y	x XOR y
1	1	0
1	0	1
0	1	1
0	0	0

Draw a logic circuit with inputs x and y whose output is always 0.

Can you use exactly 1 of the gates we've seen so far?

Fixed-width 2 binary addition *Rosen p. 251, 826*

Fixed-width addition: adding one bit at time, using the usual column-by-column and carry arithmetic, and dropping the carry from the leftmost column so the result is the same width as the summands. In many cases, this gives representation of the correct value for the sum when we interpret the summands in fixed-width binary or in 2s complement.

- Inputs x_0, y_0, x_1, y_1 represent $(x_1x_0)_{2,2}$ and $(y_1y_0)_{2,2}$
- Outputs $(x_1x_0)_{2,2} + (y_1y_0)_{2,2}$

How many bits of output should we allow for?

A. 2
B. 4
C. 6
D. 8
E. I don't know

Fixed-width w addition

Rosen p. 251, 826

$$\begin{array}{r} 1\,1\,0\,1\,0\,0 \\ +0\,0\,0\,1\,0\,1 \\ \hline 1\,1\,1\,0\,0\,1 \end{array}$$

$$\begin{array}{r} x_{w-1} \cdots x_1\ x_0 \\ +y_{w-1} \cdots y_1\ y_0 \\ \hline \end{array}$$

*Translate one symbol sum, carry to **circuit***

Input		Output
x_0	y_0	s_0
1	1	
1	0	
0	1	
0	0	

Input		Output
x_0	y_0	c_0
1	1	
1	0	
0	1	
0	0	

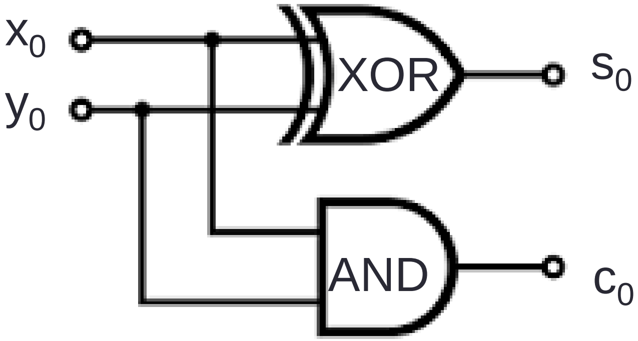
Fixed-width w addition

Rosen p. 251, 826

$$\begin{array}{r} 1\ 1\ 0\ 1\ 0\ 0 \\ +0\ 0\ 0\ 1\ 0\ 1 \\ \hline 1\ 1\ 1\ 0\ 0\ 1 \end{array}$$

$$\begin{array}{r} x_{w-1}\ \cdots\ x_1\ x_0 \\ +y_{w-1}\ \cdots\ y_1\ y_0 \\ \hline \end{array}$$

*Translate one symbol sum, carry to **circuit***



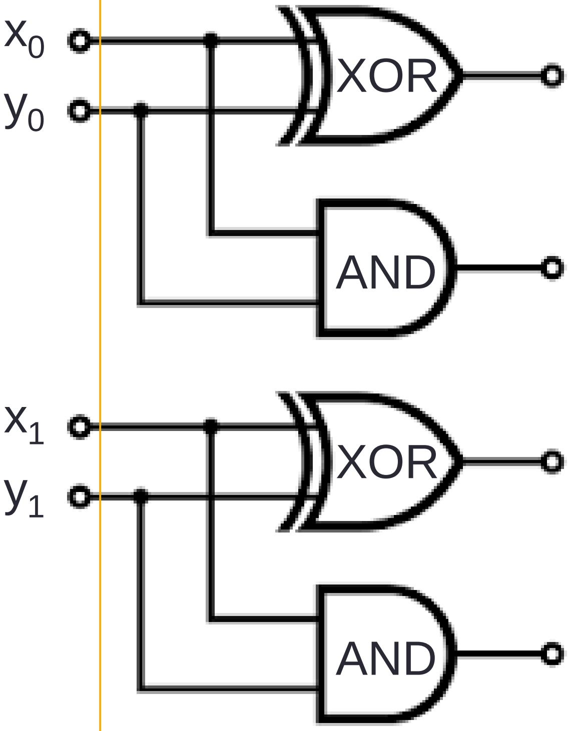
Input		Output
x_0	y_0	s_0
1	1	
1	0	
0	1	
0	0	

Input		Output
x_0	y_0	c_0
1	1	
1	0	
0	1	
0	0	

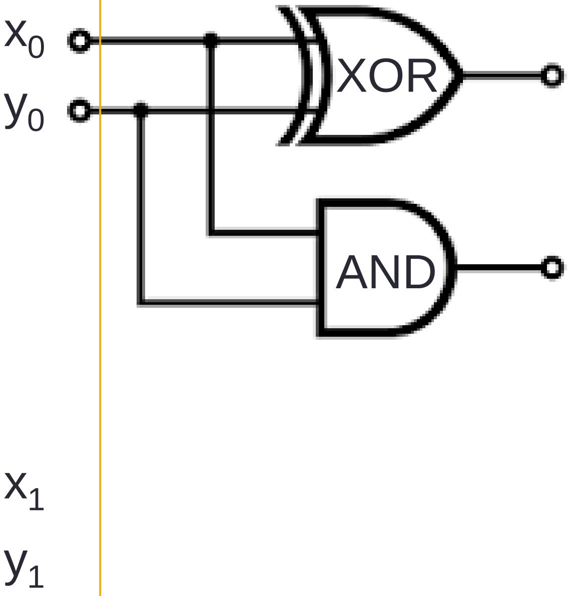
Fixed-width 2 binary addition

- Inputs x_0, y_0, x_1, y_1 represent $(x_1x_0)_{2,2}$ and $(y_1y_0)_{2,2}$
- Outputs z_0, z_1, z_2 represent $(z_2z_1z_0)_{2,3} = (x_1x_0)_{2,2} + (y_1y_0)_{2,2}$ (may require up to width 3)

Fixed-width 2 binary addition



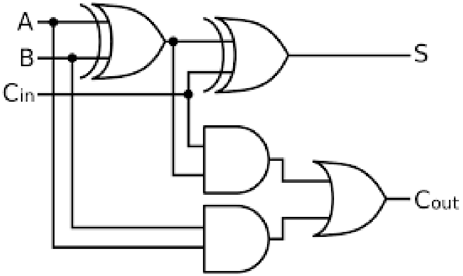
Fixed-width 2 binary addition



Logic

Rosen Section 1.1

- Use gates and circuits to express arithmetic.



- Precisely express true facts and invariant statements.

- Identify valid arguments (patterns of reasoning) that could be used in proofs.

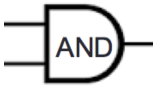
Definitions

Rosen pp. 2-4

- **Proposition**: declarative sentence that is T or F (not both)
- **Propositional variable**: variables that represent propositions.
- **Compound proposition**: new propositions formed from existing propositions using logical operators.
- **Truth table**: table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Circuits ~ Propositions

- 0 (off) ~ False
- 1 (on) ~ True



Conjunction

Input		Output
p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Exclusive or

Input		Output
p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

For next time

- Read website carefully

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

- Next pre-class reading:
  - Section 1.3 Definitions 1 and 2