base b expansion of n	**base b fixed-width w expansion of n**
For b an integer greater than 1 and n a positive integer, the **base b expansion of n** is $(a_{k-1} \cdots a_1 a_0)_b$ where k is a positive integer, $a_0, a_1, \ldots, a_{k-1}$ are nonnegative integers less than b, $a_{k-1} \neq 0$, and $n = a_{k-1} b^{k-1} + \cdots + a_1 b + a_0$	For b an integer greater than 1, w a positive integer, and n a nonnegative integer with $n < b^w$, the **base b fixed-width w expansion of n** is $(a_{w-1} \cdots a_1 a_0)_{b,w}$ where $a_0, a_1, \ldots, a_{w-1}$ are nonnegative integers less than b and $n = a_{w-1} b^{w-1} + \cdots + a_1 b + a_0$

Representing negative integers in binary: Fix a positive integer width for the representation w, $w > 1$.

	To represent a positive integer n	To represent a negative integer $-n$
Sign-magnitude	$[0a_{w-2} \cdots a_0]_{s,w}$, where $n = (a_{w-2} \cdots a_0)_{2,w-1}$ Example $n = 17$, $w = 7$:	$[1a_{w-2} \cdots a_0]_{s,w}$, where $n = (a_{w-2} \cdots a_0)_{2,w-1}$ Example $-n = -17$, $w = 7$:
2s complement	$[0a_{w-2} \cdots a_0]_{2c,w}$, where $n = (a_{w-2} \cdots a_0)_{2,w-1}$ Example $n = 17$, $w = 7$:	$[1a_{w-2} \cdots a_0]_{2c,w}$, where $2^{w-1} - n = (a_{w-2} \cdots a_0)_{2,w-1}$ Example $-n = -17$, $w = 7$:
Extra example: 1s complement	$[0a_{w-2} \cdots a_0]_{1c,w}$, where $n = (a_{w-2} \cdots a_0)_{2,w-1}$ Example $n = 17$, $w = 7$:	$[1\bar{a}_{w-2} \cdots \bar{a}_0]_{1c,w}$, where $n = (a_{w-2} \cdots a_0)_{2,w-1}$ and we define $\bar{0} = 1$ and $\bar{1} = 0$. Example $-n = -17$, $w = 7$:

Representing 0:

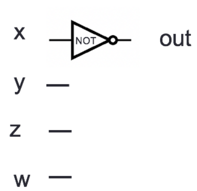
Fixed-width addition: adding one bit at time, using the usual column-by-column and carry arithmetic, and dropping the carry from the leftmost column so the result is the same width as the summands. *Does this give the right value for the sum?*

$$\begin{array}{r} (1\ 1\ 0\ 1\ 0\ 0)_{2,6} \\ +\underline{(0\ 0\ 0\ 1\ 0\ 1)_{2,6}} \end{array} \qquad \begin{array}{r} [1\ 1\ 0\ 1\ 0\ 0]_{s,6} \\ +\underline{[0\ 0\ 0\ 1\ 0\ 1]_{s,6}} \end{array} \qquad \begin{array}{r} [1\ 1\ 0\ 1\ 0\ 0]_{2c,6} \\ +\underline{[0\ 0\ 0\ 1\ 0\ 1]_{2c,6}} \end{array}$$

Extra example

$$\begin{array}{r} (1\ 1\ 0\ 1\ 0\ 0)_{2,6} \\ \times\underline{(0\ 0\ 0\ 1\ 0\ 1)_{2,6}} \end{array} \qquad \begin{array}{r} [1\ 1\ 0\ 1\ 0\ 0]_{s,6} \\ \times\underline{[0\ 0\ 0\ 1\ 0\ 1]_{s,6}} \end{array} \qquad \begin{array}{r} [1\ 1\ 0\ 1\ 0\ 0]_{2c,6} \\ \times\underline{[0\ 0\ 0\ 1\ 0\ 1]_{2c,6}} \end{array}$$

Example digital circuit:



Output when $x = 1, y = 0, z = 0, w = 1$ is _____

Output when $x = 1, y = 1, z = 1, w = 1$ is _____

Output when $x = 0, y = 0, z = 0, w = 1$ is _____