CSE 20
DISCRETE MATH

Fall 2020

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

Today's learning goals and updates

- Review -- choose your own adventure

A. Numbers (algorithms, sets, base expansions, proof strategies)
B. Movie Ratings (symbolic statements, relations and partitions)
C. RNA strands (sets and definitions)
D. Linked lists (functions and recursion and induction)
E. Sets & Cardinality (functions and proof strategies)

Numbers: algorithms

procedure *modular exponentiation*(b: integer;
$n = (a_{k-1}a_{k-2}\ldots a_1a_0)_2$, m: positive integers)
$x := 1$
$power := b \textbf{ mod } m$
for $i := 0$ **to** $k-1$
  if $a_i = 1$ **then** $x := (x \cdot power) \textbf{ mod } m$
  $power := (power \cdot power) \textbf{ mod } m$
return x $\{x \text{ equals } b^n \textbf{ mod } m\}$

$b =$ ____, $n =$ __________, $k =$ ____, $m =$ ____

i	a_i	x	$power$
		1	b **mod** $m =$
0			
1			
2			
3			

Numbers: sets

Let $W = \mathcal{P}(\{1,2,3,4,5\})$. Consider the statement

$$\forall A \in W\ \forall B \in W\ \forall C \in W\ \left((A \cap B = A \cap C)\ \rightarrow\ (B = C)\right)$$

Which of the following statements is logically equivalent to its **negation?** (Select all and only that apply.)

(a) $\forall A \in W\ \forall B \in W\ \forall C \in W\ \left((A \cap B = A \cap C)\ \wedge\ (B \neq C)\right)$

(b) $\neg \forall A \in W\ \forall B \in W\ \forall C \in W\ \neg\left((A \cap B = A \cap C)\ \wedge\ (B \neq C)\right)$

(c) $\exists A \in W\ \exists B \in W\ \exists C \in W\ \left((A \cap B = A \cap C)\ \rightarrow\ \neg(B = C)\right)$

(d) $\exists A \in W\ \exists B \in W\ \exists C \in W\ \left((A \cap B = A \cap C)\ \wedge\ \neg(B = C)\right)$

Numbers: Base expansions

Convert $(2A)_{16}$ to …

A. binary (base ?)
B. decimal (base ?)
C. octal (base ?)
D. ternary (base ?)
E. All of the above

Numbers: Every integer n >= 2 is a product of primes

Ratings

In this question, we will translate statements about movie ratings using a **new** notion of distance between pairs of n-tuples where n is a positive integer:

$$d_3(\ (x_1, \ldots, x_n), (y_1, \ldots, y_n)\ ) = \sum_{i=1}^{n} |x_i - y_i|$$

In class, we discussed the application of n-tuples to movie recommendations. Each user's ratings of movies is represented as a n-tuple (with the positive integer n being the number of movies in the database), and each component of the n-tuple is an element of the collection $\{-1, 0, 1\}$.

Assume there are five movies in the database, so that each user's ratings can be represented as a 5-tuple. We use the following definition:

- R is the set of all ratings, that is, the set of all 5-tuples where each component of the 5-tuple is an element of the collection $\{-1, 0, 1\}$.

Fill in the blank and translate:

(1) The minimum distance between any two distinct ratings is ______

(2) The maximum distance between any two distinct ratings is ______

Ratings

$$E_{proj} = \{((x_1, x_2, x_3, x_4, x_5), (y_1, y_2, y_3, y_4, y_5)) \in U \times U \mid (x_1 = y_1) \wedge (x_2 = y_2) \wedge (x_3 = y_3)\}$$

$$E_{dist} = \{(u, v) \in U \times U \mid d(u, v) \leq 2\}$$

$$E_{circ} = \{(u, v) \in U \times U \mid d((0,0,0,0,0), u) = d((0,0,0,0,0), v)\}$$

$$d((x_1, \ldots, x_n), (y_1, \ldots, y_n)) = \sum_{1 \leq i \leq n} |x_i - y_i|$$

Prove that ____ is / is not _______________ (reflexive / symmetric / transitive)

RNA strands

Each of the sets below is described using set builder notation. Rewrite them using the roster method. For example, the set described in set builder notation as

$$\{s \in S \mid \text{the leftmost base in } s \text{ is } \texttt{A} \text{ and } s \text{ has length } 2\}$$

is described using the roster method by

$$\{\texttt{AA}, \texttt{AC}, \texttt{AG}, \texttt{AU}\}.$$

Justifications aren't required for credit for this question, but it's good practice to think about how you would explain why your answer is correct.

(a) $\{s \in S \mid s \text{ has length } 2\}$

(b) $\{s \in S \mid \text{the leftmost base in } s \text{ is the same as the rightmost base in } s \text{ and } s \text{ has length } 3\}$

(c) $\{s \in S \mid \text{the bases in } s \text{ appear in alphabetical order}\}$[1]

(d) $\{s \in S \mid \text{there are twice as many } \texttt{A}\text{s as } \texttt{C}\text{s in } s \text{ and } s \text{ has length } 1\}$

RNA strands

The bases of RNA are elements of the set $B = \{\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}\}$. Certain sequences of bases serve important biological functions in translating RNA to proteins. The following recursive definition gives a special set of RNA strands that share certain biochemical properties.

Definition This set of RNA strands $\hat{S}$ is defined (recursively) by:

Basis Step:	$\texttt{AUG} \in \hat{S}$
Recursive Step:	If $s \in \hat{S}$ and $x \in R$, then $sx \in \hat{S}$

where $R = \{\texttt{UUU}, \texttt{CUC}, \texttt{AUC}, \texttt{AUG}, \texttt{GUU}, \texttt{CCU}, \texttt{GCU}, \texttt{UGG}, \texttt{GGA}\}$.

Each of the sets below is described using set builder notation. Rewrite them using the roster method. Justifications aren't required for credit for this question, but it's good practice to think about how you would explain why your answer is correct.

(a) $\{s \in \hat{S} \mid s \text{ has length less than or equal to } 5\}$

(b) $\{s \in \hat{S} \mid \text{there are twice as many } \texttt{C}\text{s as } \texttt{A}\text{s in } s \text{ and } s \text{ has length } 6\}$

Linked lists

The set of linked lists of natural numbers L is defined by:

Basis Step:	$[] \in L$
Recursive Step:	If $l \in L$ and $n \in \mathbb{N}$, then $(n, l) \in L$

Definition The function *length* : $L \to \mathbb{N}$ that computes the length of a list is:

		$length : L$	$\to \mathbb{N}$
Basis Step:		$length([])$	$= 0$
Recursive Step:	If $l \in L$ and $n \in \mathbb{N}$, then	$length((n, l))$	$= 1 + length(l)$

(a) Prove or disprove that the function *length* is onto.

(b) Prove or disprove that the function *length* is one-to-one.

Sets & Cardinality

Suppose A and B are sets and $A \subseteq B$.

A. If A is infinite then B is finite.
B. If A is countable then B is countable.
C. If B is infinite then A is finite.
D. If B is uncountable then A is uncountable.
E. None of the above.

Sets & Cardinality

Suppose A and B are sets and $A \subseteq B$. Prove that |A|≦|B|.

Diagonalization: $|\mathbb{N}| \neq |\mathcal{P}(\mathbb{N})|$

That's it

- Thank you for a great quarter

- Remember that what we covered is only the beginning. You will learn much more in future classes, using the mathematical language you learned to express many different concepts

- Good luck in the 2nd midterm and the final !