Recall that S is defined as the set of all RNA strands, strings made of the bases in $B = \{\texttt{A}, \texttt{U}, \texttt{G}, \texttt{C}\}$. Define the functions *mutation*, *insertion*, and *deletion* as described by the pseudocode below:

```
procedure mutation(b1 ··· bn : a RNA strand, k : a positive integer, b : an element of B)
for i := 1 to n
   if i = k
      ci := b
   else
      ci := bi
return c1 ··· cn {The return value is a RNA strand made of the ci values}
```

```
procedure insertion(b1 ··· bn : a RNA strand, k : a positive integer, b : an element of B)
if k > n
   for i := 1 to n
      ci := bi
   cn+1 := b
else
   for i := 1 to k − 1
      ci := bi
   ck := b
   for i := k + 1 to n + 1
      ci := bi−1
return c1 ··· cn+1 {The return value is a RNA strand made of the ci values}
```

```
procedure deletion(b1 ··· bn : a RNA strand, k : a positive integer)
if k > n
   m := n
   for i := 1 to n
      ci := bi
else
   m := n − 1
   for i := 1 to k − 1
      ci := bi
   for i := k to n − 1
      ci := bi+1
return c1 ··· cm {The return value is a RNA strand made of the ci values}
```

Mut with domain $S \times S$ is defined by, for $s_1 \in S$ and $s_2 \in S$,

$$Mut(s_1, s_2) = \exists k \in \mathbb{Z}^+ \exists b \in B(\ mutation(s_1, k, b) = s_2\ )$$

Ins with domain $S \times S$ is defined by, for $s_1 \in S$ and $s_2 \in S$,

$$Ins(s_1, s_2) = \exists k \in \mathbb{Z}^+ \exists b \in B(\ insertion(s_1, k, b) = s_2\ )$$

Del with domain $S \times S$ is defined by, for $s_1 \in S$ and $s_2 \in S$,

$$Del(s_1, s_2) = \exists k \in \mathbb{Z}^+(\ deletion(s_1, k) = s_2\ )$$

Definition: We say that a RNA strand s_1 is "within one edit" of a RNA strand s_2 to mean

$$Mut(s_1, s_2) \lor Mut(s_2, s_1) \lor Ins(s_1, s_2) \lor Ins(s_2, s_1) \lor Del(s_1, s_2) \lor Del(s_2, s_1)$$

$within1_{TF}$: _____ $\to$ _____ $within1_{\mathcal{P}}$: _____ $\to$ _____

$within1_{TF}(s_1, s_2)$ = ____________________ $within1_{\mathcal{P}}(s_1)$ = ____________________

$W_1 = \{$ ______________________________ $\}$

Definition: When A and B are sets, we say any subset of $A \times B$ is a **binary relation**. There are other ways to represent a relation R

- A function f_{TF} : __________ $\to$ __________ with $f_{TF}(\quad) =$ _____________
- A function $f_{\mathcal{P})}$:________$\to$________ with $f_{\mathcal{P}}(\quad) =$ _____________

Definition: When A is a set, we say any subset of $A \times A$ is a (binary) **relation** on A.

Definition: Let $R_{(\mathbf{mod}\ n)}$ be the set of all pairs of integers (a, b) such that $(a\ \mathbf{mod}\ n = b\ \mathbf{mod}\ n)$. Then a is **congruent to** b **mod** n means $(a, b) \in R_{(\mathbf{mod}\ n)}$. A common notation is to write this as $a \equiv b(\mathbf{mod}\ n)$.

Some example elements of $R_{(\mathbf{mod}\ 4)}$ are: ____

Definition: (*Rosen 9.1*) A relation R on a set A is called **reflexive** means $(a, a) \in R$ for every element $a \in A$. A relation R on a set A is called **symmetric** means $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$. A relation R on a set A is called **transitive** means whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Relation	Reflexive? (why/why not)	Symmetric? (why/why not)	Transitive? (why/why not)
W_1			
$R_{(\mathbf{mod}\ 4)}$			

Definition: (*Rosen 9.5*) A relation is an **equivalence relation** means it is reflexive, symmetric, and transitive.

Definition: (*Rosen 9.5*) An **equivalence class** of an element $a \in A$ for an equivalence relation R on the set A is the set $\{s \in A | (a, s) \in R\}$. We write this as $[a]_R$.

Some examples of elements of $[5]_{R_{(\mathbf{mod}\ 4)}}$ are: ____________________________________

Some examples of elements of $[9]_{R_{(\mathbf{mod}\ 4)}}$ are: ____________________________________

Some examples of elements of $[6]_{R_{(\mathbf{mod}\ 4)}}$ are: ____________________________________

Definition: A **partition** of a set A is a set of non-empty, disjoint subsets $A_1, A_2, \cdots, A_n$ such that $A_1 \cup A_2 \cup \cdots \cup A_n = A$.

We can partition the set of integers using equivalence classes of $R_{(\mathbf{mod}\ 4)}$ using: __________________________