CSE 20
DISCRETE MATH

Fall 2020

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

Today's learning goals

- Define (binary) relations and give examples.
- Define equivalence using relations and give examples.

Application: genomics

An idea important to genomics – are two RNA (or DNA) strands "close" ? For today – consider all RNA strands that are no more than "1 edit" away from a strand. An "edit" means applying one of the procedures *mutation* or *insertion* or *deletion.*

Mutation: $Mut: S \times \mathbb{N} \times B \to S$

$Mut(s, k, b)$ – replace in s at index k the current base with b

Insertion: $Ins: S \times \mathbb{N} \times B \to S$

$Ins(s, k, b)$ – insert in s at index k a new base b

Deletion: $Del: S \times \mathbb{N} \to S$

$Del(s, k)$ – delete in s current base at index k

Application: genomics

An idea important to genomics – are two RNA (or DNA) strands "close" ? For today – consider all RNA strands that are no more than "1 edit" away from a strand. An "edit" means applying one of the procedures *mutation* or *insertion* or *deletion.*

Mut with domain $S \times S$ is defined by, for $s_1 \in S$ and $s_2 \in S$,

$$Mut(s_1, s_2) = \exists k \in \mathbb{Z}^+ \exists b \in B(\ mutation(s_1, k, b) = s_2\ )$$

Ins with domain $S \times S$ is defined by, for $s_1 \in S$ and $s_2 \in S$,

$$Ins(s_1, s_2) = \exists k \in \mathbb{Z}^+ \exists b \in B(\ insertion(s_1, k, b) = s_2\ )$$

Del with domain $S \times S$ is defined by, for $s_1 \in S$ and $s_2 \in S$,

$$Del(s_1, s_2) = \exists k \in \mathbb{Z}^+(\ deletion(s_1, k) = s_2\ )$$

Definition: We say that a RNA strand s_1 is "within one edit" of a RNA strand s_2 to mean

$$Mut(s_1, s_2) \lor Mut(s_2, s_1) \lor Ins(s_1, s_2) \lor Ins(s_2, s_1) \lor Del(s_1, s_2) \lor Del(s_2, s_1)$$

Application: genomics

Which of these strands are within 1 edit of `UCUCA`?

A: `UCCCA`

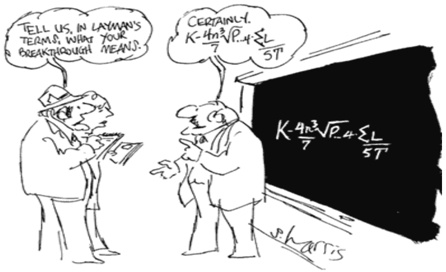
B: `UCCA`

C: `UCUCA`

D: `AUCUCA`

E: All of the above

Multiple Representations



Application: genomics

What data structure could we use to represent "RNA strands that differ by no more than 1 edit"?

A: A function with domain *S* and codomain *S*

B: A function with domain *S* and codomain *P(S)*

C: A function with domain *S x S* and codomain *{T, F}*

D: A set that is a subset of *S x S*

E: All of the above

UCUCA	UCUCA	UCUCA	UCUCA
|	|	|	| |
UCUCAU	UCUC	UCGCA	UCGCU

Application: genomics

$$within1_{TF} : \_\_\_\_\_ \to \_\_\_\_\_$$

$$within1_{TF}(s_1, s_2) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$within1_{\mathcal{P}} : \_\_\_\_ \to \_\_\_\_$$

$$within1_{\mathcal{P}}(s_1) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Let the **binary relation** W_1 be the set of all pairs of RNA strands that are within one edit of one another.

$W_1 =$

Relations

New idea: A **binary relation** R from A to B is a subset of A x B.

Sometimes we will say "a **(binary) relation on A**" where A is a set, which means a relation from A to A, which means a subset of A x A.

The set of pairs of RNA strands that are 1 edit away from each other is an example of a **binary relation on S**, where S is the set of RNA strands.

Which ideas have we seen that are relations already?

An Important Integer Relation(ship)

Definition: Let $R_{(\textbf{mod } n)}$ be the set of all pairs of integers (a, b) such that $(a \textbf{ mod } n = b \textbf{ mod } n)$.

We say a is **congruent to** b **mod** n means $(a, b) \in R_{(\textbf{mod } n)}$. A common notation is to write this as $a \equiv b(\textbf{mod } n)$.

Some example elements of $R_{(\textbf{mod } 4)}$ are:

Properties of Relations

Definition: (*Rosen 9.1*) A relation R on a set A is called **reflexive** means $(a, a) \in R$ for every element $a \in A$.

Which of these relations are reflexive?

A. W_1

B. $R_{(\mathbf{mod}\ 4)}$

C. Both satisfy the property

D. Neither satisfies the property

Properties of Relations

Definition: (*Rosen 9.1*) A relation R on a set A is called **symmetric** means $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$.

Which of these relations are symmetric?

A. W_1

B. $R_{(\textbf{mod } 4)}$

C. Both satisfy the property

D. Neither satisfies the property

Properties of Relations

Definition: (*Rosen 9.1*) A relation R on a set A is called **transitive** means whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Which of these relations are transitive?

A. W_1

B. $R_{(\mathbf{mod}\ 4)}$

C. Both satisfy the property

D. Neither satisfies the property

Properties of Relations

Definition: (*Rosen 9.5*) A relation is an **equivalence relation** means it is reflexive, symmetric, and transitive.

Definition: (*Rosen 9.5*) An **equivalence class** of an element $a \in A$ for an equivalence relation R on the set A is the set $\{s \in A | (a, s) \in R\}$. We write this as $[a]_R$.

Some examples of elements of $[5]_{R_{(\mathbf{mod}\ 4)}}$ are: ____________________

Some examples of elements of $[9]_{R_{(\mathbf{mod}\ 4)}}$ are: ____________________

Some examples of elements of $[6]_{R_{(\mathbf{mod}\ 4)}}$ are: ____________________

Properties of Relations

Definition: A **partition** of a set A is a set of non-empty, disjoint subsets $A_1, A_2, \cdots, A_n$ such that $A_1 \cup A_2 \cup \cdots \cup A_n = A$.

We can partition the set of integers using equivalence classes of $R_{(\mathbf{mod}\ 4)}$ using:

Main takeaway

- Equivalence relations let us group “similar” elements
- A partition gives rise to an equivalence relation and vice versa