$\mathbb{Z}$	The set of integers	$\{\ldots,-2,-1,0,1,2,\ldots\}$
$\mathbb{Z}^+$	The set of positive integers	$\{1,2,\ldots\}$
$\mathbb{N}$	The set of nonnegative integers	$\{0,1,2,\ldots\}$
$\mathbb{Q}$	The set of rational numbers	$\left\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z} \text{ and } q \neq 0\right\}$
$\mathbb{R}$	The set of real numbers	

$$\_\_ \subsetneq \_\_ \subsetneq \_\_ \subsetneq \_\_ \subsetneq \_\_$$

Extra example: Define two infinite sets of numbers such that neither is a subset of the other.

Definition (Rosen p139): Let D and C be nonempty sets. A **function** f from D (domain) to C (codomain) is an assignment of one element of C to each element of D.

Definition (Rosen p141): A function $f : D \to C$ is **one-to-one** (or injective) means for every a, b in the domain D, if $f(a) = f(b)$ then $a = b$.

Definition: For sets A, B, we say that **the cardinality of A is no bigger than the cardinality of B**, and write $|A| \leq |B|$, to mean there is a one-to-one function with domain A and codomain B.

Let S_2 be the set of RNA strands of length 2.

Statement	True/False , justification
$\|\{1,2,3\}\| \leq \|\{\texttt{A},\texttt{U},\texttt{G},\texttt{C}\}\|$	
$\|\{\texttt{A},\texttt{U},\texttt{G},\texttt{C}\}\| \leq \|S_2\|$	
$\|\{\texttt{A},\texttt{U},\texttt{G},\texttt{C}\} \times \{\texttt{A},\texttt{U},\texttt{G},\texttt{C}\}\| \leq \|S_2\|$	

Definition (Rosen p143): A function $f : D \to C$ is **onto** (or surjective) means for every b in the codomain, there is an element a in the domain with $f(a) = b$.

Formally, $f : D \to C$ is onto means ____________________.

Definition: For sets A, B, we say that **the cardinality of A is no smaller than the cardinality of B**, and write $|A| \geq |B|$, to mean there is an onto function with domain A and codomain B.

Let S_2 be the set of RNA strands of length 2.

Statement	True/False , justification
$\lvert\{\texttt{A}, \texttt{U}, \texttt{G}, \texttt{C}\}\rvert \geq \lvert\{1, 2, 3\}\rvert$	
$\lvert S_2\rvert \geq \lvert\{\texttt{A}, \texttt{U}, \texttt{G}, \texttt{C}\}\rvert$	
$\lvert S_2\rvert \geq \lvert\{\texttt{A}, \texttt{U}, \texttt{G}, \texttt{C}\} \times \{\texttt{A}, \texttt{U}, \texttt{G}, \texttt{C}\}\rvert$	

Definition (Rosen p144): A function $f : D \to C$ is a **bijection** means that it is both one-to-one and onto. The **inverse** of a bijection $f : D \to C$ is the function $g : C \to D$ such that $g(b) = a$ iff $f(a) = b$.

Extra example: Prove that for any nonempty sets A, B, there is a bijection with domain A and codomain B if and only if there is a bijection with domain B and codomain A.

For nonempty sets A, B we say

$|A| \leq |B|$ means there is a one-to-one function with domain A, codomain B

$|A| \geq |B|$ means there is an onto function with domain A, codomain B

$|A| = |B|$ means there is a bijection with domain A, codomain B

Cantor-Schroder-Bernstein Theorem: For all nonempty sets,

$$|A| = |B| \quad \text{if and only if} \quad (|A| \leq |B| \text{ and } |B| \leq |A|) \quad \text{if and only if} \quad (|A| \geq |B| \text{ and } |B| \geq |A|)$$