Proof by Structural Induction (Rosen 5.3 p354) To prove a universal quantification over a recursively defined set:

Basis Step: Show the statement holds for elements specified in the basis step of the definition.

Recursive Step: Show that if the statement is true for each of the elements used to construct new elements in the recursive step of the definition, the result holds for these new elements.

Theorem: A robot on an infinite 2-dimensional integer grid starts at $(0, 0)$ and at each step moves to diagonally adjacent grid point. This robot can / cannot (*circle one*) reach $(1, 0)$.

Definition The set of positions the robot can visit P is defined by:

Basis Step:	$(0, 0) \in P$	
Recursive Step:	If $(x, y) \in P$, then	are also in P

Lemma: $\forall (x, y) \in P((x + y \text{ is an even integer}))$

Proof of theorem using lemma: To show is $(1, 0) \notin P$. Rewriting the lemma to explicitly restrict the domain of the universal, we have $\forall (x, y)$ ($(x, y) \in P \rightarrow (x + y$ is an even integer)). Since the universal is true, ($(1, 0) \in P \rightarrow (1 + 0$ is an even integer)) is a true statement. Evaluating the conclusion of this conditional statement: By definition of long division, since $1 = 0 \cdot 2 + 1$ (where $0 \in \mathbb{Z}$ and $1 \in \mathbb{Z}$ and $0 \leq 1 < 2$ mean that 0 is the quotient and 1 is the remainder), $1 \textbf{ mod } 2 = 1$ which is not 0 so the conclusion is false. A true conditional with a false conclusion must have a false hypothesis. Thus, $(1, 0) \notin P$, QED. □

Proof of lemma by structural induction:

Basis Step

Recursive Step. Consider arbitrary $(x, y) \in P$. To show is:

$$(x + y \text{ is an even integer}) \rightarrow (\text{sum of coordinates of next position is even integer})$$

Assume **as the induction hypothesis, IH** that:

"New"! Proof by Mathematical Induction (Rosen 5.1 p329)
To prove a universal quantification over the set of all integers greater than or equals some base integer b:

> **Basis Step**: Show the statement holds for b.
>
> **Recursive Step**: Consider an arbitrary integer n greater than or equal to b, assume (as the **induction hypothesis**) that the property holds for n, and use this and other facts to prove that the property holds for $n+1$.

Recall that the set of linked lists of natural numbers L	Recall that length of a linked list of natural numbers L, $length : L \to \mathbb{N}$ is defined by:
Basis Step: $[] \in L$ Recursive Step: If $l \in L$ and $n \in \mathbb{N}$ then $(n, l) \in L$	Basis step: $length([]) = 0$ Recursive step: If $l \in L$ and $n \in \mathbb{N}$ then $length((n, l)) = 1 + length(l)$

Prove or disprove: $\forall n \in \mathbb{N}\ \exists l \in L\ (\ length(l) = n\ )$

Extra example: Functions on $\mathbb{Z}^+$ *i.e. domain and codomain both equal* $\mathbb{Z}^+$

Name	**Definition**
Exponent function	Basis Step: If $n = 1$ then $2^n = 2$ Recursive Step: If $n \in \mathbb{Z}^+$, then $2^{n+1} = 2 \cdot 2^n$
Factorial function	Basis Step: If $n = 1$ then $n! = 1$ Recursive Step: If $n \in \mathbb{Z}^+$, then $(n+1)! = (n+1) \cdot n!$

Prove or disprove: $\exists b \in \mathbb{Z}^+\ \forall n \in \mathbb{Z}^{\geq b}\ (2^n < n!)$

By mathematical induction:

Basis Step

Recursive Step. Consider . To show is:

$\to$

Assume **as the induction hypothesis, IH** that: