Term	**Definition**	**Examples**
set	an unordered collection of elements	
set equality	When A and B are sets, $A = B$ means $\forall x(x \in A \leftrightarrow x \in B)$	$\{43, 7, 9\} = \{7, 43, 9, 7\}$ $\left\{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\right\} = \left\{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
subset	When A and B are sets, $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$	
proper subset	When A and B are sets, $A \subsetneq B$ means $(A \subseteq B) \wedge (A \neq B)$	

Prove or **disprove** the following claims:

Claim: $\{\texttt{A}, \texttt{C}, \texttt{U}, \texttt{G}\} \subseteq \{\texttt{AA}, \texttt{AC}, \texttt{AU}, \texttt{AG}\}$	Claim: $\{4, 6\} \subseteq \{n \textbf{ mod } 10 \mid \exists c \in \mathbb{Z}(n = 4c)\}$
Claim: The empty set is a proper subset of every set.	Claim: For some set B, $\emptyset \in B$.

Proof by universal generalization: To prove that $\forall x\, P(x)$ is true, we can take an arbitrary element e from the domain and show that $P(e)$ is true, without making any assumptions about e other than that it comes from the domain.

Evidence for conjunction being true or false:
To prove that $p \wedge q$ is true, have two subgoals: subgoal (1) prove p is true; and, subgoal (2) prove q is true.
To prove that $p \wedge q$ is false, it's enough to prove that p is false.
To prove that $p \wedge q$ is false, it's enough to prove that q is false.

New! Proof by Cases: To prove q, if we know that $p_1 \vee p_2$ is true, and we can show that $(p_1 \rightarrow q)$ is true and we can show that $(p_2 \rightarrow q)$, then we can conclude q is true. Sec 1.8 p92

Term	Definition	Examples
Cartesian product	When A and B are sets, $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$	$\{43, 9\} \times \{9, A\} =$ $\mathbb{Z} \times \emptyset =$
union	When A and B are sets, $A \cup B = \{x \mid x \in A \vee x \in B\}$	$\{43, 9\} \cup \{9, A\} =$ $\mathbb{Z} \cup \emptyset =$
intersection	When A and B are sets, $A \cap B = \{x \mid x \in A \wedge x \in B\}$	$\{43, 9\} \cap \{9, A\} =$ $\mathbb{Z} \cap \emptyset =$
set difference	When A and B are sets, $A - B = \{x \mid x \in A \wedge x \notin B\}$	$\{43, 9\} - \{9, A\} =$ $\mathbb{Z} - \emptyset =$
disjoint sets	sets A and B are disjoint means $A \cap B = \emptyset$	$\{43, 9\}, \{9, A\}$ are not disjoint $\mathbb{Z}, \emptyset$ are disjoint
power set	When S is a set, $\mathcal{P}(S) = \{X \mid X \subseteq S\}$	$\mathcal{P}(\{43, 9\}) =$ $\mathcal{P}(\emptyset) =$

Let $W = \mathcal{P}(\{1, 2, 3, 4, 5\}) =$ ______________________________

Prove or **disprove**: $\forall A \in W \, \forall B \in W \; (A \subseteq B \; \rightarrow \; \mathcal{P}(A) \subseteq \mathcal{P}(B))$

Extra example: **Prove** or **disprove**: $\forall A \in W \, \forall B \in W \; (\mathcal{P}(A) = \mathcal{P}(B) \; \rightarrow \; A = B)$

Extra example: **Prove** or **disprove**: $\forall A \in W \, \forall B \in W \, \forall C \in W \; (A \cup B = A \cup C \; \rightarrow \; B = C)$

Proof of conditional by direct proof: To prove that the conditional statement $p \rightarrow q$ is true, we can assume p is true and use that assumption to show q is true.
New! Proof of Conditional by Contrapositive Proof: To prove that the implication $p \rightarrow q$ is true, we can assume q is false and use that assumption to show p is also false. Sec 1.7 p83