CSE 20
DISCRETE MATH

Fall 2020

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

Overall strategy

Understand the statement
- Logical structure
- Relevant definitions

Do you believe the statement?
- Try some small examples that illustrate relevant claims

Map out possible proof strategies
- For each strategy: what can we assume? What evidence do we need?
- Start with simplest strategies, move to more complicated if/when we get stuck

Work to prove / disprove statement **(sometimes in parallel…)**

Recap

- To prove that $\forall x P(x)$ is true, use exhaustion or universal generalization.
- To prove that $\forall x P(x)$ is false, use a counterexample.
- To prove that $\exists x P(x)$ is true, use a witness.
- To prove that $\exists x P(x)$ is false, write universal statement that is logically equivalent to its negation and then prove it true using universal generalization.
- Direct proof: To prove that HYP→CONC, assume the HYP and work to prove that CONC is true.

Today's plan: practice these strategies in a new context, and add to them

Definitions

Rosen Sections 2.1, 2.2

A **set** is an ________________ collection of elements

Set **equality** A = B means $\forall x(x \in A \leftrightarrow x \in B)$

$A \subseteq B$ means A is a **subset** of B, aka B is a **superset** of A

Formally: $\forall x(x \in A \rightarrow x \in B)$

$A \subsetneq B$ means A is a **proper subset** of B, aka B is a **proper superset** of A

Formally: $(A \subseteq B) \wedge (A \neq B)$

subset	When A and B are sets, $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$	
proper subset	When A and B are sets, $A \subsetneq B$ means $(A \subseteq B) \wedge (A \neq B)$	
empty set	The set that has no elements	$\{\}$, $\emptyset$

Which of the following is true?

A. $\{\mathtt{A}, \mathtt{C}, \mathtt{U}, \mathtt{G}\} \subseteq \{\mathtt{AA}, \mathtt{AC}, \mathtt{AU}, \mathtt{AG}\}$

B. $\{4, 6\} \subseteq \{n \textbf{ mod } 10 \mid \exists c \in \mathbb{Z}(n = 4c)\}$

C. The empty set is a proper subset of every set.

D. For some set B, $\emptyset \in B$.

E. None of the above.

Prove or **disprove** the following claims:

Claim: $\{\texttt{A}, \texttt{C}, \texttt{U}, \texttt{G}\} \subseteq \{\texttt{AA}, \texttt{AC}, \texttt{AU}, \texttt{AG}\}$	Claim: $\{4, 6\} \subseteq \{n \textbf{ mod } 10 \mid \exists c \in \mathbb{Z}(n = 4c)\}$

Prove or **disprove** the following claims:

Claim: The empty set is a proper subset of every set.	Claim: For some set B, $\emptyset \in B$.

Term	Definition
Cartesian product	When A and B are sets, $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$
union	When A and B are sets, $A \cup B = \{x \mid x \in A \vee x \in B\}$
intersection	When A and B are sets, $A \cap B = \{x \mid x \in A \wedge x \in B\}$
set difference	When A and B are sets, $A - B = \{x \mid x \in A \wedge x \notin B\}$

Which of the following sets are equal?

#1 $\{43, 9\} \times \{9, A\}$

#2 $\{43, 9\} \cup \{9, A\}$

#3 $\{43, 9\} \cap \{9, A\}$

#4 $\{43, 9\} - \{9, A\}$

A. #1, #2
B. #2, #3
C. #3, #4
D. #2, #3
E. None of the above

power set	When S is a set, $\mathcal{P}(S) = \{X \mid X \subseteq S\}$	$\mathcal{P}(\{43, 9\}) =$ $\mathcal{P}(\emptyset) =$

Let $W = \mathcal{P}(\{1, 2, 3, 4, 5\}) =$ ____

Prove or **disprove**: $\forall A \in W \, \forall B \in W \; (A \subseteq B \;\rightarrow\; \mathcal{P}(A) \subseteq \mathcal{P}(B))$

Prove or **disprove**: $\forall A \in W \, \forall B \in W \;\; (\mathcal{P}(A) = \mathcal{P}(B) \;\rightarrow\; A = B)$

Proofs

To prove that the conditional

$$p \rightarrow q$$

is true, we can assume p is true and use that assumption to show q is true.

New!

Proofs

To prove that the implication

$$p \rightarrow q$$

is true, we can assume q is false and use that assumption to show p is false

New!

Proofs

To prove that q holds when we know

$$p_1 \vee p_2$$

is true, we can show two conditional statements:

Goal 1: $(p_1 \to q)$

Goal 2: $(p_2 \to q)$

Then conclude q

New!

For next time

- Read website carefully

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

Pre-reading for induction:

Section 5.3 Definition of Structural Induction (p 354)