

Recurring example: encoding and decoding information

Consider the following algorithm to introduce redundancy in a string of 0s and 1s.

Create redundancy by repeating each bit three times

```

1 procedure redun3( $a_{k-1} \cdots a_0$ : a binary string)
2 for  $i := 0$  to  $k-1$ 
3    $c_{3i} := a_i$ 
4    $c_{3i+1} := a_i$ 
5    $c_{3i+2} := a_i$ 
6 return  $c_{3k-1} \cdots c_0$ 

```

Decode sequence of bits using majority rule on consecutive three bit sequences

```

1 procedure decode3( $c_{3k-1} \cdots c_0$ : a binary string whose length is an integer multiple of 3)
2 for  $i := 0$  to  $k-1$ 
3   if exactly two or three of  $c_{3i}, c_{3i+1}, c_{3i+2}$  are set to 1
4      $a_i := 1$ 
5   else
6      $a_i := 0$ 
7 return  $a_{k-1} \cdots a_0$ 

```

Give a recursive definition of the set of outputs of the *redun3* procedure, *Out*,

Basis step: _____

Recursive step: _____

Consider the message $m = 0001$ so that the sender calculates $\text{redun3}(m) = \text{redun3}(0001) = 000000000111$.

Introduce ____ errors into the message so that the signal received by the receiver is _____ but the receiver is still able to decode the original message.

Challenge: what is the biggest number of errors you can introduce?

Building a circuit for line 3 in *decode* procedure: given three input bits, we need to determine whether the majority is a 0 or a 1.

c_{3i}	c_{3i+1}	c_{3i+2}	a_i
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

Circuit

Recurring example: RNA strands

Recall that S is defined as the set of all RNA strands, strings made of the bases in $B = \{A, U, G, C\}$. Consider the following predicates:

$Rep_{AC}(s, n) =$ “The RNA strand has at least n repeats of AC”

$Rep_{AAC}(s, n) =$ “The RNA strand has at least n repeats of AAC”

Translate to a formal statement, write its negation, and determine which is true.

If A has at least one repeat of AC then it has at least one repeat of AAC.

Negation is: _____

Extra example: Is exclusive or associative?

It is sufficient for a strand to have at least 3 repeats of AAC for it to have at least 2 repeats of AC.

Negation is: _____

For at least one strand, it has at least 2 repeats of AC exactly when it has at least 2 repeats of AAC.

Negation is: _____