Recall the predicate $F(a,b) = \exists c \in \mathbb{Z}\ (b = ac)$ is a predicate over the domain $\mathbb{Z}^{\neq 0} \times \mathbb{Z}$. In English, $F(a,b)$ evaluates to T means a is a nonzero integer, b is an integer, and a is a factor of b. An equivalent definition is that $F(a,b) = T$ exactly when b **mod** $a = 0$.

Definition (Rosen p. 257): An integer p greater than 1 is called **prime** means the only positive factors of p are 1 and p. We write $Pr(x)$ to indicate that an positive integer x is prime. A positive integer that is greater than 1 and is not prime is called composite.

Claim: The statement "There are three consecutive positive integers that are prime." is True / False

Hint: These numbers would be of the form $p, p+1, p+2$ (where p is a positive integer).

Proof: We need to show ____

Claim: The statement "There are three consecutive odd positive integers that are prime." is True / False

Hint: These numbers would be of the form $p, p+2, p+4$ (where p is an odd positive integer).

Proof: We need to show ____

Proof of universal by exhaustion: To prove that $\forall x\, P(x)$ is true when P has a finite domain, evaluate the predicate at **each** domain element to confirm that it is always T.

Proof by universal generalization: To prove that $\forall x\, P(x)$ is true, we can take an arbitrary element e from the domain and show that $P(e)$ is true, without making any assumptions about e other than that it comes from the domain.

To prove that $\exists x P(x)$ is **false**, write the universal statement that is logically equivalent to its negation and then prove it true using universal generalization.

To prove that $p \wedge q$ is true, have two subgoals: subgoal (1) prove p is true; and, subgoal (2) prove q is true.

To prove that $p \wedge q$ is false, it's enough to prove that p is false.
To prove that $p \wedge q$ is false, it's enough to prove that q is false.

Each Netflix user's viewing history can be represented as a n-tuple indicating their preferences about movies in the database, where n is the number of movies in the database. Each element in the n-tuple indicates the user's rating of the corresponding movie: 1 indicates the person liked the movie, -1 that they didn't, and 0 that they didn't rate it one way or another. Consider a four movie database. We denote the set of possible ratings $\{-1,0,1\} \times \{-1,0,1\} \times \{-1,0,1\} \times \{-1,0,1\}$ as R_4. We have the function

$$d_{1,4} : R_4 \times R_4 \to \mathbb{N} \text{ where } d_{1,4}(\ (x_1,x_2,x_3,x_4),(y_1,y_2,y_3,y_4)\ ) = \max_{1\leq i\leq 4} |x_i - y_i|$$

$$d_{2,4} : R_4 \times R_4 \to \mathbb{N} \text{ where } d_{2,4}(\ (x_1,x_2,x_3,x_4),(y_1,y_2,y_3,y_4)\ ) = \sqrt{\sum_{i=1}^{4}(x_i - y_i)^2}$$

Consider the following predicates:

Predicate	Domain	Example domain element where predicate is T	Example domain element where predicate is F
$d_{1,4}(r_1,r_2) < d_{2,4}(r_1,r_2)$	$R_4 \times R_4$		
$\exists r_0 \in R_4\ (\ d_{1,4}(r,r_0) = 1\ )$	R_4		

Claim: $\forall r_1 \in R_4\ \forall r_2 \in R_4\ (\ r_1 = r_2 \to \neg\,(d_{1,4}(r_1,r_2) < d_{2,4}(r_1,r_2))\ )$

In English: ____

Extra example: Is the claim equivalent to $\forall r \in R_4\ (\ \neg\,(d_{1,4}(r,r) < d_{2,4}(r,r))\ )$?

$(p \to q) \equiv \neg(p \wedge \neg q)$ $\qquad \neg(p \wedge q) \equiv \neg p \vee \neg q$ $\qquad q \vee \neg p \equiv p \to q$ $\qquad \neg\exists x\, P(x) \equiv \forall x\, \neg(P(x))$

New! Proof of conditional by direct proof: To prove that the conditional statement $p \to q$ is true, we can assume p is true and use that assumption to show q is true.