CSE 20
DISCRETE MATH

Fall 2020

http://cseweb.ucsd.edu/classes/fa20/cse20-a/

Today's goals

- Add to our repertoire of proof strategies
- Identify the main connective of a proposition and associated proof strategies
- Determine whether a proposition is true or false using valid reasoning (proofs)

Review

Proof of universal by exhaustion: To prove that $\forall x\, P(x)$ is true when P has a finite domain, evaluate the predicate at **each** domain element to confirm that it is always T.

Proof by universal generalization: To prove that $\forall x\, P(x)$ is true, we can take an arbitrary element e from the domain and show that $P(e)$ is true, without making any assumptions about e other than that it comes from the domain.

To prove that $\exists x P(x)$ is **false**, write the universal statement that is logically equivalent to its negation and then prove it true using universal generalization.

Review

Recall the predicate $F(a,b) = \exists c \in \mathbb{Z}\ (b = ac)$ is a predicate over the domain $\mathbb{Z}^{\neq 0} \times \mathbb{Z}$. In English, $F(a,b)$ evaluates to T means a is a nonzero integer, b is an integer, and a is a factor of b. An equivalent definition is that $F(a,b) = T$ exactly when $b \textbf{ mod } a = 0$.

Definition (Rosen p. 257): An integer p greater than 1 is called **prime** means the only positive factors of p are 1 and p. We write $Pr(x)$ to indicate that an positive integer x is prime. A positive integer that is greater than 1 and is not prime is called composite.

Formally:

Practically, use "Trial Division": **THEOREM 2** If n is a composite integer, then n has a prime divisor less than or equal to $\sqrt{n}$.

Extra example: prove that Trial Division works. (Section 4.3, page 258)

Translate

"There are three consecutive positive integers that are prime."

Translates to:

A. $\exists p_1 \in \mathbb{Z}^+ \; \exists p_2 \in \mathbb{Z}^+ \; \exists p_3 \in \mathbb{Z}^+ \; (\; Pr(p_1) \wedge Pr(p_2) \wedge Pr(p_3) \;)$

B. $\exists p \in \mathbb{Z}^+ \; (\; Pr(p) \rightarrow (\; Pr(p+1) \wedge Pr(p+2) \;) \;)$

C. $\exists p \in \mathbb{Z}^+ \; (\; Pr(p) \wedge Pr(p+1) \wedge Pr(p+2) \;)$

D. More than one of the above

E. None of the above

Translate

“There are three consecutive positive integers that are prime.”

Translates to:

“There are three consecutive **odd** positive integers that are prime.”

Translates to:

True or False?

Which of the following statements is true?

A. “There are three consecutive positive integers that are prime.”

B. “There are three consecutive odd positive integers that are prime.”

C. Both of these statements

D. None of the above

Claim: The statement “There are three consecutive positive integers that are prime.” is True / False

Claim: The statement "There are three consecutive odd positive integers that are prime." is True / False

Each Netflix user's viewing history can be represented as a n-tuple indicating their preferences about movies in the database, where n is the number of movies in the database. Each element in the n-tuple indicates the user's rating of the corresponding movie: 1 indicates the person liked the movie, -1 that they didn't, and 0 that they didn't rate it one way or another. Consider a four movie database. We denote the set of possible ratings $\{-1,0,1\} \times \{-1,0,1\} \times \{-1,0,1\} \times \{-1,0,1\}$ as R_4. We have the function

$$d_{1,4} : R_4 \times R_4 \to \mathbb{N} \text{ where } d_{1,4}(\ (x_1,x_2,x_3,x_4),(y_1,y_2,y_3,y_4)\ ) = \max_{1 \leq i \leq 4} |x_i - y_i|$$

$$d_{2,4}(\ (x_1,x_2,x_3,x_4),(y_1,y_2,y_3,y_4)\ ) = \sqrt{\sum_{i=1}^{4} (x_i - y_i)^2}$$

Consider the following predicates:

Predicate	Domain	Example domain element where predicate is T	Example domain element where predicate is F
$d_{1,4}(r_1,r_2) < d_{2,4}(r_1,r_2)$	$R_4 \times R_4$		
$\exists r_0 \in R_4\ (\ d_{1,4}(r,r_0) = 1\ )$	R_4		

Claim: $\forall r_1 \in R_4 \; \forall r_2 \in R_4 \; (\; r_1 = r_2 \to \neg (d_{1,4}(r_1, r_2) < d_{2,4}(r_1, r_2)) \;)$

In English: ____________________

Claim: $\forall r_1 \in R_4 \; \forall r_2 \in R_4 \; (\; r_1 = r_2 \to \neg (d_{1,4}(r_1, r_2) < d_{2,4}(r_1, r_2)) \;)$

In English: ____________________

To prove that the conditional

p→q

is true, we can assume p is true and use that assumption to show q is true.

New!

Recap

- Valid proof strategy depends on the logical structure of statement being proved.
- Keywords in proofs:
  - Need to show
  - Assume
  - Let … be an arbitrary element of ….
  - Towards a proof by …..