- To prove that $\forall x P(x)$ is true: ____
- To prove that $\forall x P(x)$ is false: ____
- To prove that $\exists x P(x)$ is true: ____
- To prove that $\exists x P(x)$ is false: ____

Some sets of numbers

$\mathbb{N}$	The set of natural numbers	$\{0, 1, 2, 3, \ldots\}$	*Recursively defined by* Basis step: Recursive step:
$\mathbb{Z}$	The set of integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$	*Recursively defined by* Basis step: Recursive step:
$\mathbb{Z}^+$	The set of positive integers	$\{1, 2, 3, \ldots\}$	*Set builder notation definition is* $\{x \in \mathbb{N} \mid x > 0\} = \{x \in \mathbb{Z} \mid x > 0\}$
$\mathbb{Z}^{\neq 0}$	The set of nonzero integers		*Set builder notation definition is* $\{x \in \mathbb{Z} \mid (x < 0 \vee x > 0)\}$

Factoring

Definition (Rosen p. 238): When a and b are integers and a is nonzero, a **divides** b means there is an integer c such that $b = ac$.

Terminology: a is a **factor** of b, a is a **divisor** of b, b is a **multiple** of a, $a|b$

Symbolically, $F(a, b) =$ ______________________________ and is a predicate over the domain _______________

Claim: Every nonzero integer is a factor of itself.

Proof:

Proof of universal by exhaustion: To prove that $\forall x\, P(x)$ is true when P has a finite domain, evaluate the predicate at **each** domain element to confirm that it is always T.

New! Proof by universal generalization: To prove that $\forall x\, P(x)$ is true, we can take an arbitrary element e from the domain and show that $P(e)$ is true, without making any assumptions about e other than that it comes from the domain.

Claim: The statement "There is a nonzero integer that does not divide its square" is True / False

Circle one

Proof:

Definition (Rosen p. 257): An integer p greater than 1 is called **prime** means the only positive factors of p are 1 and p. A positive integer that is greater than 1 and is not prime is called composite.

A formal definition of the predicate Pr over the domain $\mathbb{Z}$ which evaluates to T exactly when the input is prime is:

Claim: 1 is not prime.

Proof:

Claim: 4 is not prime.

Proof:

$(p \to q) \equiv \neg(p \wedge \neg q)$ $\qquad$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\qquad$ $q \vee \neg p \equiv p \to q$ $\qquad$ $\neg \exists x\, P(x) \equiv \forall x\, \neg(P(x))$

To prove that $\exists x P(x)$ is **false**, write the universal statement that is logically equivalent to its negation and then prove it true using universal generalization.

To prove that $p \wedge q$ is true, have two subgoals: subgoal (1) prove p is true; and, subgoal (2) prove q is true.

To prove that $p \wedge q$ is false, it's enough to prove that p is false.
To prove that $p \wedge q$ is false, it's enough to prove that q is false.