Definition (Rosen p123): The **Cartesian product** of the sets A and B, $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. That is: $A \times B = \{(a, b) \mid (a \in A) \wedge (b \in B)\}$. The Cartesian product of the sets $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \times \cdots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, ..., a_n)$, where a_i belongs to A_i for $i = 1, 2, \ldots, n$. That is, $A_\times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \ldots, n\}$

Recall: Each RNA strand is a string whose symbols are elements of the set $B = \{\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}\}$. The **set of all RNA strands** is called S. The function *rnalen* that computes the length of RNA strands in S is:

$$\begin{array}{lll} & & rnalen : S \rightarrow \mathbb{Z}^+ \\ \text{Basis Step:} & \text{If } b \in B \text{ then} & rnalen(b) = 1 \\ \text{Recursive Step:} & \text{If } s \in S \text{ and } b \in B\text{, then} & rnalen(sb) = 1 + rnalen(s) \end{array}$$

A function *basecount* that computes the number of a given base b appearing in a RNA strand s is:

$$\begin{array}{llll} & & basecount : S \times B & \rightarrow \mathbb{N} \\ \text{Basis Step:} & \text{If } b_1 \in B, b_2 \in B & basecount(b_1, b_2) & = \begin{cases} 1 & \text{when } b_1 = b_2 \\ 0 & \text{when } b_1 \neq b_2 \end{cases} \\ \text{Recursive Step:} & \text{If } s \in S, b_1 \in B, b_2 \in B & basecount(sb_1, b_2) & = \begin{cases} 1 + basecount(s, b_2) & \text{when } b_1 = b_2 \\ basecount(s, b_2) & \text{when } b_1 \neq b_2 \end{cases} \end{array}$$

L with domain $S \times \mathbb{Z}^+$ is defined by, for $s \in S$ and $n \in \mathbb{Z}^+$,

$$L(s, n) = \begin{cases} T & \text{if } rnalen(s) = n \\ F & \text{otherwise} \end{cases}$$

Element where L evaluates to T: _______

Element where L evaluates to F: _______

BC with domain __________ is defined by, for $s \in S$ and $b \in B$ and $n \in \mathbb{N}$,

$$BC(s, b, n) = \begin{cases} T & \text{if } basecount(s, b) = n \\ F & \text{otherwise} \end{cases}$$

Element where BC evaluates to T: ________

Element where BC evaluates to F: _________

Notation: for a predicate P with domain $X_1 \times \cdots \times X_n$ and a n-tuple $(x_1, \ldots, x_n)$ with each $x_i \in X$, we write $P(x_1, \ldots, x_n)$ to mean $P(\ (x_1, \ldots, x_n)\ )$.

$\exists t \; BC(t)$ In English: ______________________________

Witness that proves this existential quantification is true: ____________

$\forall (s,b,n) \; (\; BC(s,b,n) \;)$ In English: ______________________________

Counterexample that proves this universal quantification is false: ____________

New predicates from old $BC(s,b,n)$ means $basecount(s,b) = n$.

Predicate	Domain	Example domain element where predicate is T
$basecount(s,b) = 3$		
$basecount(s,\texttt{A}) = n$		
$\exists n \in \mathbb{N} \; (basecount(s,b) = n)$		
$\forall b \in B \; (basecount(s,b) = 1)$		

Alternating quantifiers

$$\forall s \exists n \; BC(s,\texttt{A},n)$$

In English: ______________________________

$$\exists n \forall s \; BC(s,\texttt{U},n)$$

In English: ______________________________

Evaluate each quantified statement as T or F.

$\forall s \; \forall b \; \exists n \; BC(s,b,n)$	$\forall s \; \forall n \; \exists b \; BC(s,b,n)$	$\forall b \; \forall n \; \exists s \; BC(s,b,n)$
$\exists s \; \forall b \; \exists n \; BC(s,b,n)$	$\forall s \; \exists n \; \forall b \; BC(s,b,n)$	$\exists b \; \exists n \; \forall s \; BC(s,b,n)$

Extra example: Write the negation of each of the statements above, and use De Morgan's law to find a logically equivalent version where the negation is applied only to the BC predicate (not next to a quantifier).