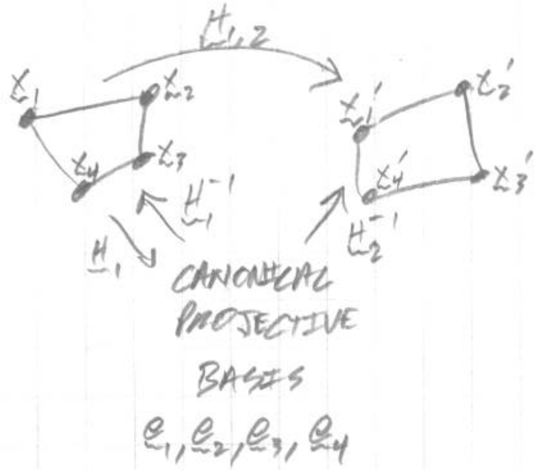
2D PROJECTIVE TRANSFORMATION, 4 POINT



$\underline{x}_i' = \underline{H}_{1,2}\, \underline{x}_i$ FOR $i = 1, 2, 3, 4$

$\underline{e}_i = \underline{H}_1\, \underline{x}_i$

$\underline{x}_i' = \underline{H}_2^{-1}\, \underline{e}_i$

$\underline{x}_i' = \underline{H}_2^{-1}\, \underline{H}_1\, \underline{x}_i$

$\underline{x}_i' = \underline{H}_2^{-1} (\underline{H}_1^{-1})^{-1}\, \underline{x}_i$

$\underline{x}_i' = \underline{H}_{1,2}\, \underline{x}_i$

WHERE $\underline{H}_{1,2} = \underline{H}_2^{-1} (\underline{H}_1^{-1})^{-1}$

HOW TO SOLVE FOR $\underline{H}_1^{-1}$ AND $\underline{H}_2^{-1}$?

2D PROJECTIVE TRANSFORMATION, 4 POINT TO STANDARD BASIS OF $\mathbb{P}^2$

STANDARD BASIS OF $\mathbb{P}^2$

$$\underset{\sim}{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \underset{\sim}{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \underset{\sim}{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \underset{\sim}{e}_4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

THE POINTS AT INFINITY ALONG EACH AXIS — THE ORIGIN — THE "UNIT POINT"

4 POINTS CAN BE MAPPED TO STANDARD BASIS

$$[\underset{\sim}{e}_1 | \underset{\sim}{e}_2 | \underset{\sim}{e}_3 | \underset{\sim}{e}_4] = \underset{\sim}{H} [s_1 \underset{\sim}{x}_1 | s_2 \underset{\sim}{x}_2 | s_3 \underset{\sim}{x}_3 | s_4 \underset{\sim}{x}_4]$$ DIVIDE BY s_4

$$[\underset{\sim}{e}_1 | \underset{\sim}{e}_2 | \underset{\sim}{e}_3 | \underset{\sim}{e}_4] = \underset{\sim}{H} [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3 | \underset{\sim}{x}_4]$$ WHERE $\lambda_1 = s_1/s_4$, $\lambda_2 = s_2/s_4$, AND $\lambda_3 = s_3/s_4$ $(s_4/s_4 = 1)$

FIRST 3 COLUMNS

$$[\underset{\sim}{e}_1 | \underset{\sim}{e}_2 | \underset{\sim}{e}_3] = \underset{\sim}{H} [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

$$\underset{\sim}{I} = \underset{\sim}{H} [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

$$\underset{\sim}{H}^{-1} = [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

LAST COLUMN

$$\underset{\sim}{e}_4 = \underset{\sim}{H} \underset{\sim}{x}_4$$

$$\underset{\sim}{H}^{-1} \underset{\sim}{e}_4 = \underset{\sim}{x}_4$$

$$[\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3] \underset{\sim}{e}_4 = \underset{\sim}{x}_4$$

$$[\underset{\sim}{x}_1 | \underset{\sim}{x}_2 | \underset{\sim}{x}_3] \underset{\sim}{\lambda} = \underset{\sim}{x}_4$$ WHERE $\underset{\sim}{\lambda} = (\lambda_1, \lambda_2, \lambda_3)^T$

SOLVE FOR $\underset{\sim}{\lambda}$

$$\underset{\sim}{H}^{-1} = [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

NOTE: THIS RESULT SCALES TO n DIMENSIONS