CSE 105, Fall 2019 - Homework 6

Due: Monday 11/25 midnight

Instructions Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Sections 3.1, 3.2, 4.1

Key Concepts TM, Decidability
Recall the terminology for describing Turing Machines as per the class slides:

- **Formal definition**: set of states, input alphabet, tape alphabet, transition function, state, accept state, reject state. A state diagram is sufficient for defining the transition function.
- **Implementation-level definition**: English prose to describe Turing machine head movements relative to contents of tape.
- **High-level description**: Description of algorithm, without implementation details of machine. As part of this description, can "call" and run another TM as a subroutine.

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**Problem 1 (10 points)**

Let \( L = \{w \in \{a,b\}^* | |w|^a = 1 + |w|^b \} \) where \( |w|^a \) denotes the number of occurrences of \( a \) in \( w \) and \( |w|^b \) denotes the number of occurrences of \( b \) in \( w \).

Provide formal description of a Turing machine \( M \) which decides \( L \).

The transition function can be represented as a state diagram.

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**Problem 2 (10 points)**

The Fibonacci numbers, commonly denoted \( F_n \), form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. That is, \( F_0 = 0 \), \( F_1 = 1 \) and \( F_n = F_{n-1} + F_{n-2} \), for \( n > 1 \).

The beginning of the sequence is thus: \( 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots \)

\( L = \{1^n | n \text{ is a Fibonacci number and } n>0 \} \). Construct a deterministic Turing Machine that decides \( L \). Provide an implementation level description for \( L \).

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**Problem 3 (10 points)**

Prove the following statements:

a) The class of decidable languages is closed under concatenation
b) The class of recognizable languages is closed under Kleene star
Problem 4 (10 points)

Prove that a language is turing-decidable if and only if some enumerator enumerates the strings of this language in lexicographic order.

Problem 5 (10 points)

A queue automaton is like a PDA except that the stack is replaced by a queue. A queue is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we’ll call it a push) adds a symbol to the left-hand end of the queue and each read operation (we’ll call it a pull) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at anytime. Show that a language can be recognized by a deterministic queue automaton if and only if the language is Turing-recognizable.