

Order Notation Cheat Sheet

It was brought to my attention that some class members have not seen formal definitions of order notation before. Since I assumed that these had been seen in my review, I have drawn up this cheat sheet on formal definitions. Please note that the definition numbering in this document is my own, and is in no way related to the book or material presented in class.

Definition 1.1 *O*-notation

For a given function $g(n)$, $O(g(n))$ is defined as:

$$O(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq c \cdot g(n), \text{ for all } n \geq n_0 \}$$

When we say $f(n) = O(g(n))$, or $f(n)$ is $O(g(n))$, what we actually mean is that $f(n)$ is an element of the set $O(g(n))$. **This is true for all order notation.**

Intuitively, this means that $g(n)$ is an upper bound on $f(n)$ (analogous to $g(n) \geq f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = O(n^4)$, or $f(n)$ is $O(n^4)$. This is true because $f(n) \leq cg(n)$, for $c = 10$, $n_0 = 1$. Also, note that $f(n) = O(n^3)$.

Definition 1.2 *o*-notation

For a given function $g(n)$, $o(g(n))$ is defined as:

$$o(g(n)) = \{ f(n) : \text{for any positive constant } c, \text{ there exists a constant } n_0 \text{ such that } 0 \leq f(n) < cg(n), \text{ for all } n \geq n_0 \}$$

Intuitively, this means that $g(n)$ is an upper bound of $f(n)$ that is not tight (analogous to $g(n) > f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = o(n^4)$ or $f(n)$ is $o(n^4)$. Note that $f(n)$ is **not** $o(n^3)$.

Definition 1.3 Θ -notation

For a given function $g(n)$, $\Theta(g(n))$ is defined as:

$$\Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1g(n) \leq f(n) \leq c_2g(n), \text{ for all } n \geq n_0 \}$$

Intuitively, this means that $g(n)$ is roughly equivalent to $f(n)$ (analogous to $g(n) \approx f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = \Theta(n^3)$ or $f(n)$ is $\Theta(n^3)$.

Definition 1.4 Ω -notation

For a given function $g(n)$, $\Omega(g(n))$ is defined as:

$$\Omega(g(n)) = \{ f(n) : \text{there exist positive constants } c_1 \text{ and } n_0 \text{ such that } 0 \leq c_1g(n) \leq f(n), \text{ for all } n \geq n_0 \}$$

Intuitively, this means that $g(n)$ is a lower bound on $f(n)$ (analogous to $g(n) \leq f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = \Omega(n^3)$ or $f(n)$ is $\Omega(n^3)$. Also, we have that $f(n) = \Omega(n^2)$.

Definition 1.5 ω -notation

For a given function $g(n)$, $\omega(g(n))$ is defined as:

$$\omega(g(n)) = \{ f(n) : \text{for any positive constant } c, \text{ there exists a constant } n_0 \text{ such that } 0 \leq cg(n) < f(n), \text{ for all } n \geq n_0 \}$$

Intuitively, this means that $g(n)$ is lower bound of $f(n)$ that is not tight (analogous to $g(n) < f(n)$ for real numbers).

For example, if $f(n) = n^3 + 4n^2 + 5$, then we can say $f(n) = \omega(n^2)$ or $f(n)$ is $\omega(n^2)$. However, $f(n)$ is **not** $\omega(n^3)$.