CSE 101 Fall 2018
Homework 4
Due: Tuesday, October 30, 2018 at 11:59pm

INSTRUCTIONS

This homework assignment may be completed in groups of size 1-4. The solutions must be **typed** (using a computer.) Figures and graphs can be hand drawn. For algorithm descriptions we require a high-level English description AND an implementation description. If you find it necessary to also include a pseudo code to help understand your description then feel free to include it.

Please refer to the course page for requirements in writing up answers for algorithm questions.

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1. Consider the problem where you are finding a spanning tree of a graph, but instead of minimizing the total weight, you want an MST that minimizes the maximum weight of an edge in the graph. Show that Kruskal's algorithm also is optimal for this objective function.(25 points)

2. You are given an undirected connected graph $G = (V, E)$ with positive edge weights, and a minimum spanning tree $T = (V, E')$ with respect to these weights; you may assume G and T are given as adjacency lists. Now suppose the weight of a particular edge $e \in E$ is modied from $w(e)$ to a new value $\hat{w}(e)$. You wish to quickly update the minimum spanning tree T to reect reflect this change, without recomputing the entire tree from scratch. There are four cases. In each case give a linear-time algorithm for updating the tree. (no correctness proof necessary. Please justify the runtime.)(25 points)
   (a) $e \notin E'$ and $\hat{w}(e) > w(e)$
   (b) $e \notin E'$ and $\hat{w}(e) < w(e)$
   (c) $e \in E'$ and $\hat{w}(e) < w(e)$
   (d) $e \in E'$ and $\hat{w}(e) > w(e)$

3. In this problem, we will develop a new algorithm for finding minimum spanning trees. It is based upon the following property:

   Let G be an undirected connected graph with at least one cycle.
   Pick a cycle in G, and let e be the heaviest edge in that cycle. Then there is a minimum spanning tree that does not contain e.

   (a) (8 points) Prove this property.
   (b) (7 points) Here is the new MST algorithm.

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   procedure new_MST(G = (V, E); an undirected connected graph with positive
   edge weights {w_e}.)
   sort the edges according to their weights
   for each edge e ∈ E in decreasing order of weights:
     if e is part of a cycle of G:
       G = G − e (that is, remove e from G)
   return G
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   Prove the correctness of this algorithm.
   (c) (5 points) On each iteration, the algorithm must check if e is part of a cycle of G. Give a linear time algorithm to do this.

(d) (5 points) Calculate the runtime of this algorithm in terms of $|E|$.

4. Spectrum: You want to create a scientific laboratory capable of monitoring any frequency in the electromagnetic spectrum between L and H. You have a list of possible monitoring technologies, T_i, $i = 1, ..n$, each with an interval $[l_i, h_i]$ of frequencies that it can be used to monitor. You want to pick as few as possible technologies that together cover the interval $[L, H]$.

   Candidate Greedy Strategy I: First, buy the technology that covers the longest sub-interval within (L, H). (i.e.,The longest interval (l, h), but not including the sub-intervals (l, L) and (H, h) outside the interval we need covering.) At each subsequent step, buy the technology that covers the largest total length that is still uncovered.

   Candidate Greedy Strategy II: Look at all the technologies with $l \leq L$. Of these, buy the one $T_i = (l_i, h_i)$ with the largest value of h_i. Repeat the process to cover the remaining interval, (h_i, H).
   (8 points for arguing the optimal algorithm, 9 points for proof of correctness, 8 points for time analysis, space analysis and efficiency.)