CSE291 Convex Optimization: Problem Statement

CK Cheng

Dept. of Computer Science and Engineering

University of California, San Diego

Outlines

- General Convex Problem Formats
- Convex Sets
  - Specification
  - Sets and Definitions
- Convex Functions
- Convex Optimization Problems

General Formats

min $f_0(x)$

subject to $f_i(x) \leq b_i$, i=1, …, m,

where functions $f_0,\ldots,f_m$: $R^n \rightarrow R$ are convex, i.e.

- $f_i(\alpha x+\beta y) \leq \alpha f_i(x)+\beta f_i(y)$
- for all x, y ϵ R^n and all α, β ϵ R with $\alpha+\beta=1$, $\alpha \geq 0$, $\beta \geq 0$.

Examples:

General Formats

min $f_0(x)$

subject to $f_i(x) \leq b_i$, i=1, …, m,

- f_0 is a convex function
- $\{x | f_i(x) \leq b_i\}$ is a convex set for all i=1, …,m

Convex set C: for all x, yϵ C

- $\alpha x+\beta y \epsilon$ C, for all $\alpha+\beta=1$, and $\alpha, \beta \geq 0$.

Examples:

Outlines

- General Convex Problem Formats
- Convex Sets
  - Specification

Sets and Definitions

- Affine Sets, Cones, Convex Hulls
- Hyperplanes and Half Spaces
- Polyhedra
- Matrix Positive Semidefinite Cones
- Dual Cones

Convex Set Specification

We can describe the convex sets using

- Implicit Expression (equations)
- Explicit Expression (enumerations)

Examples:

Explicit Expression: Examples

$$\{\theta_1 u_1+\theta_2 u_2+\ldots+\theta_k u_k \mid \theta_1+\theta_2+\ldots+\theta_k=1,\ \theta_i \geq 0, \text{ for all } i\}$$

Sets and Definitions

- Affine Sets, Cones, Convex Hulls
- Hyperplanes and Half Spaces
- Polyhedra (poly + hedron)
- Matrix Positive Semidefinite Cones
- Dual Cones

Examples