Matrix-Based Transforms and The Wavelet Transform

Image Processing

CSE 166

Lecture 13

Reading

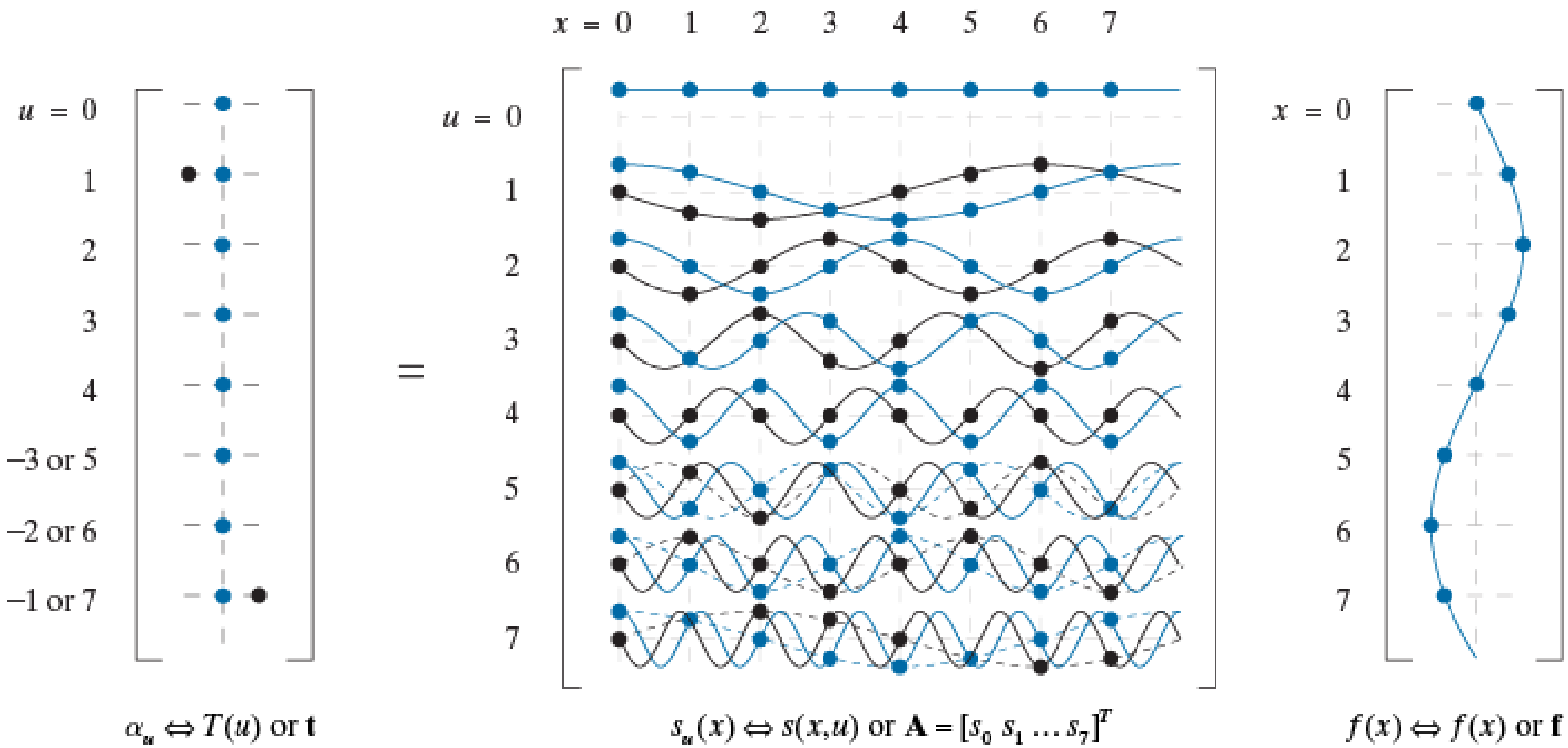
- Digital Image Processing, 4th edition
  - Chapter 6: Wavelet and Other Image Transforms
    - Sections 6.2—6.10

Matrix-based transforms

- Discrete Fourier transform (DFT)
- Discrete Hartley transform (DHT)
- Discrete cosine transform (DCT)
- Discrete sine transform (DST)
- Walsh-Hadamard (WHT)
- Slant (SLT)
- Haar (HAAR)
- Daubechies (DB4)
- Biorthogonal B-spline (BIOR3.1)

Basis vectors of matrix-based 1D transforms

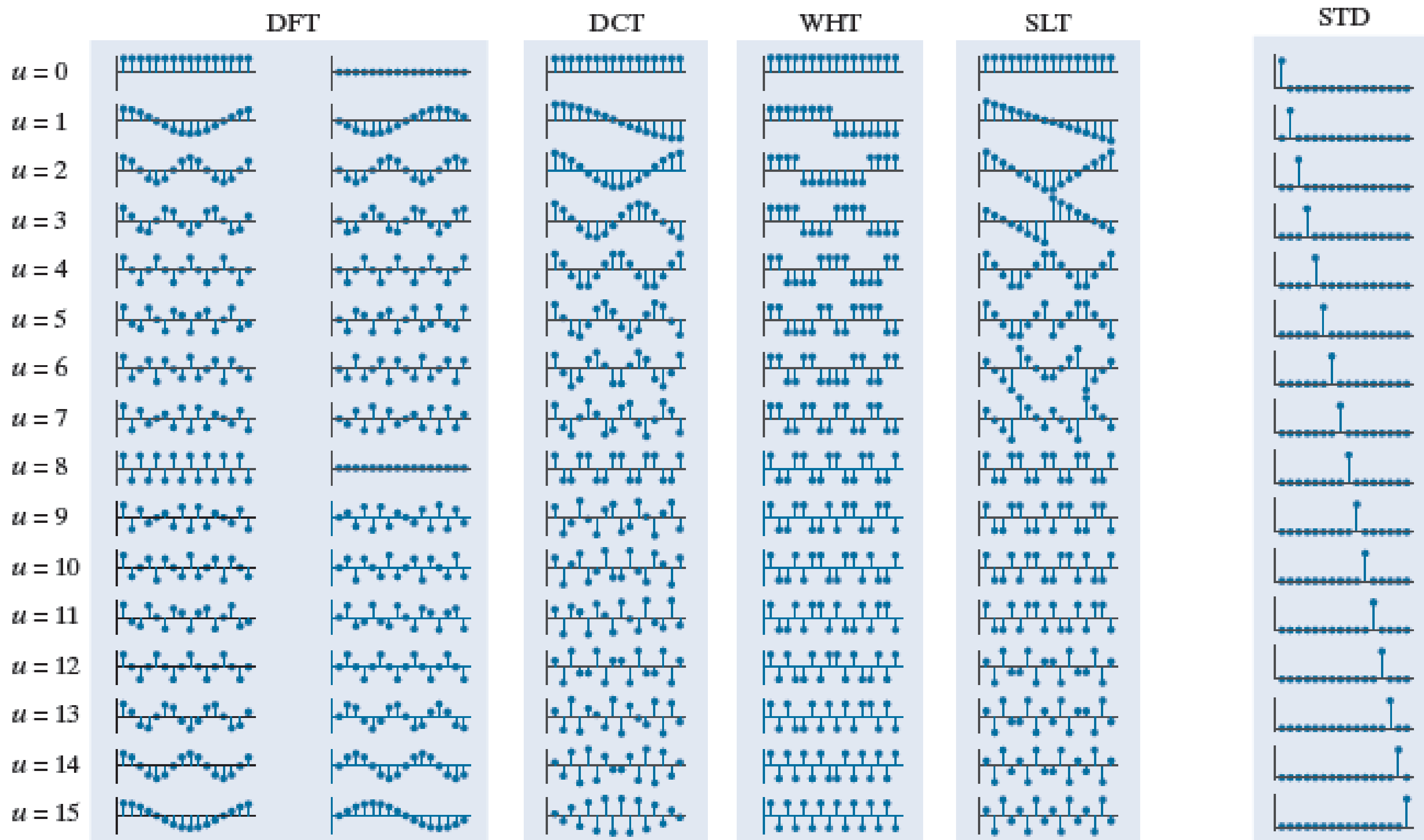
Example: DFT of $f(x) = \sin(2\pi x)$, N = 8



real part + imaginary part

Basis vectors of matrix-based 1D transforms

N = 16



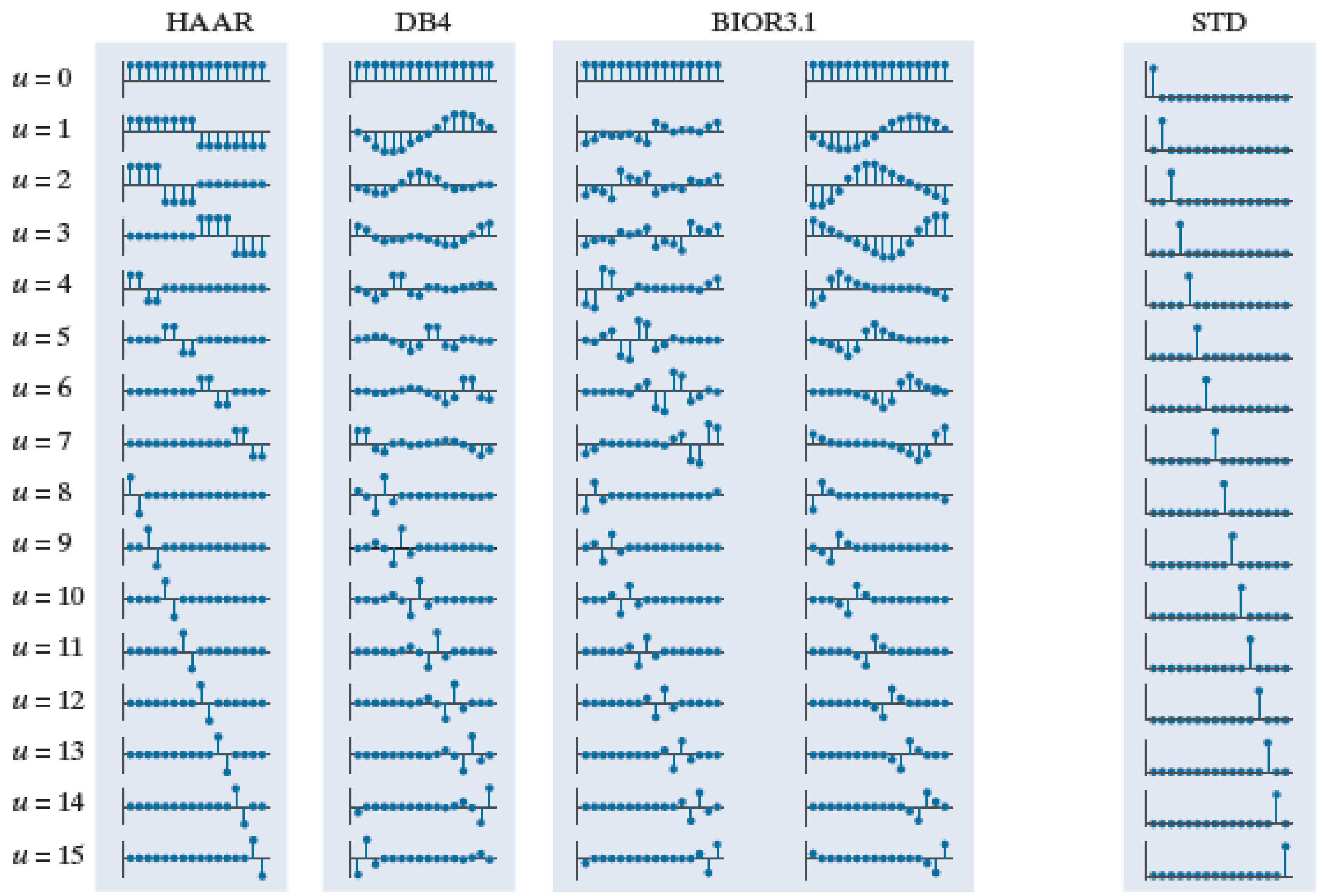
real part

imaginary part

Standard basis (for reference)

Basis vectors of matrix-based 1D transforms

N = 16

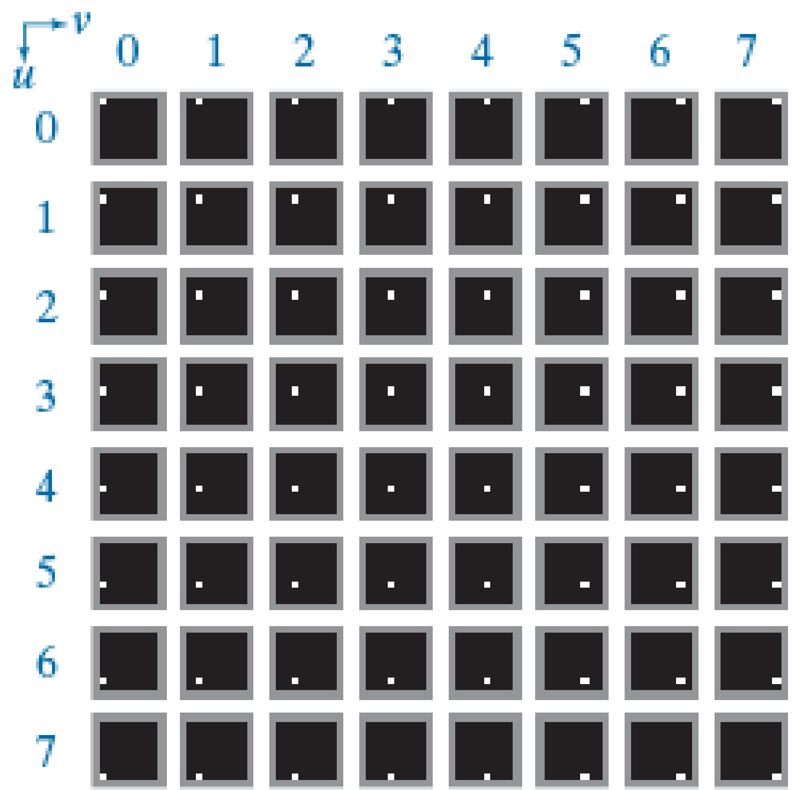


basis dual

Standard basis (for reference)

Basis images of matrix-based 2D transforms

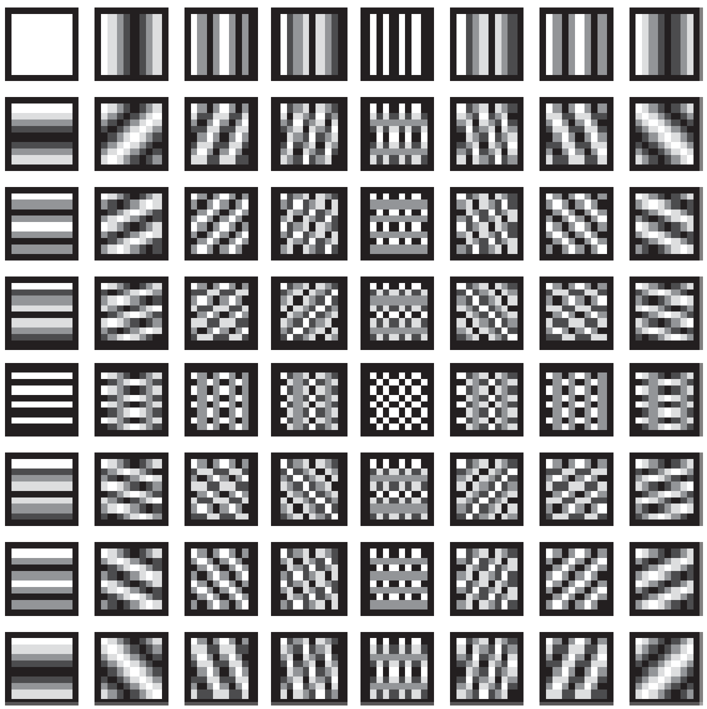
Standard basis images (for reference)



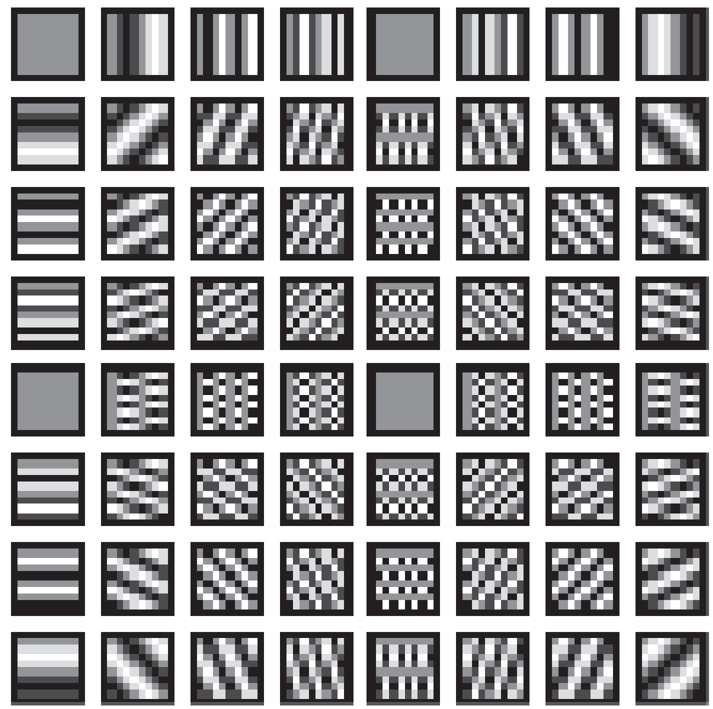
8-by-8
array of
8-by-8
basis images

Basis images of matrix-based 2D transforms

Discrete Fourier transform (DFT) basis images



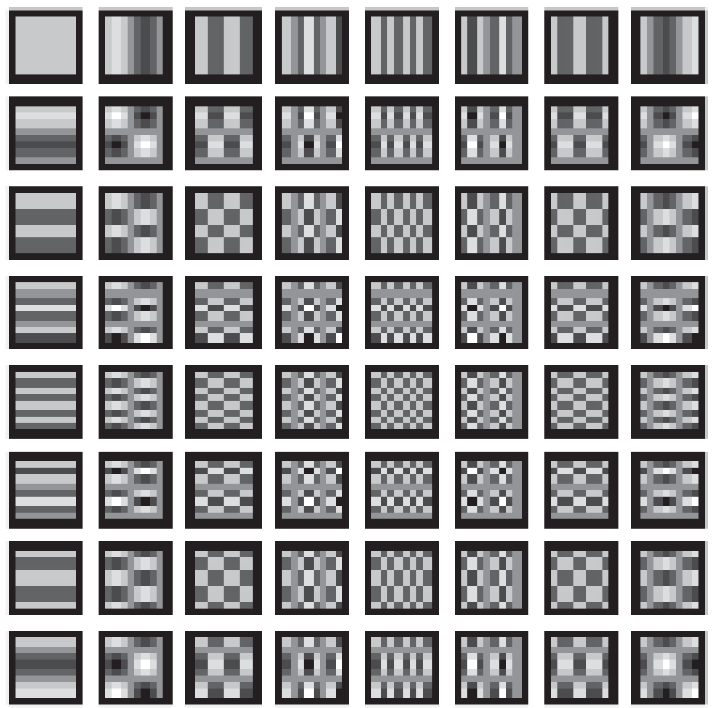
real part



imaginary part

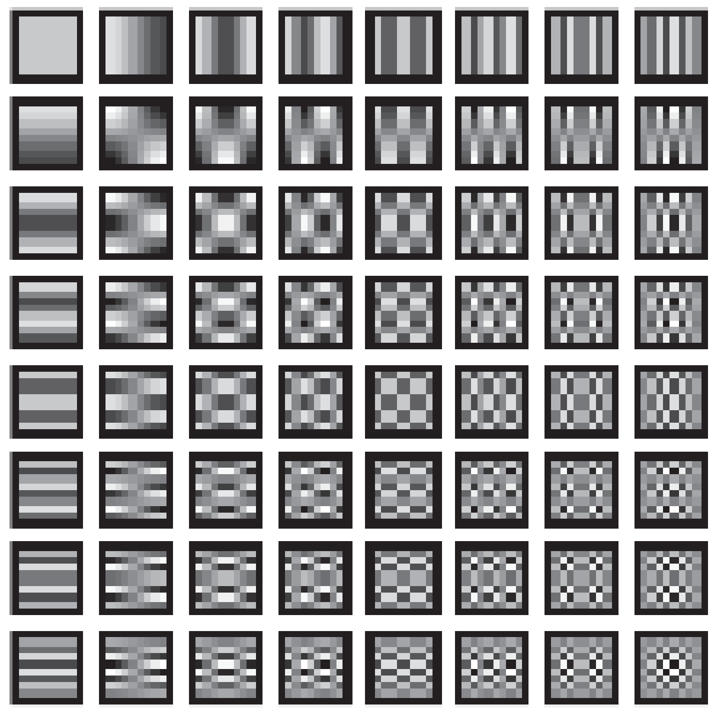
Basis images of matrix-based 2D transforms

Discrete Hartley transform (DHT) basis images



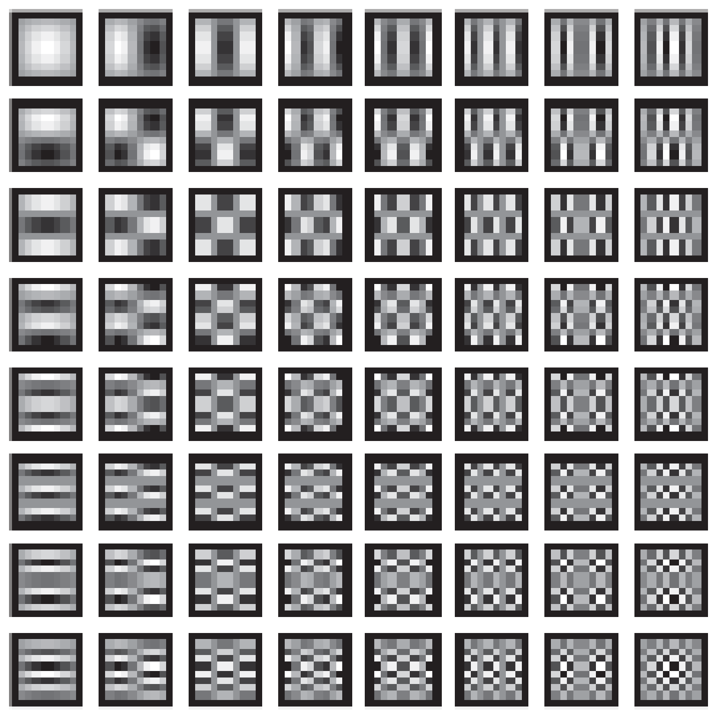
Basis images of matrix-based 2D transforms

Discrete cosine transform (DCT) basis images



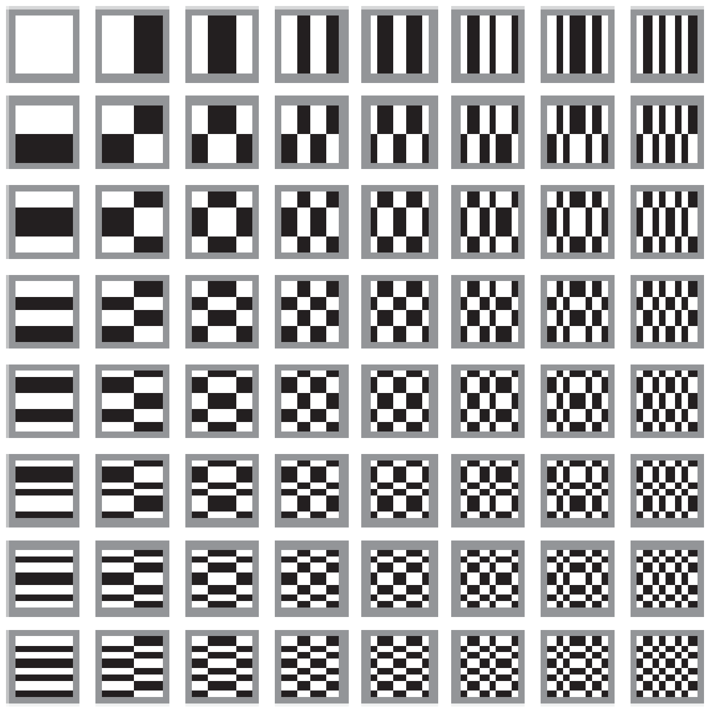
Basis images of matrix-based 2D transforms

Discrete sine transform (DST) basis images



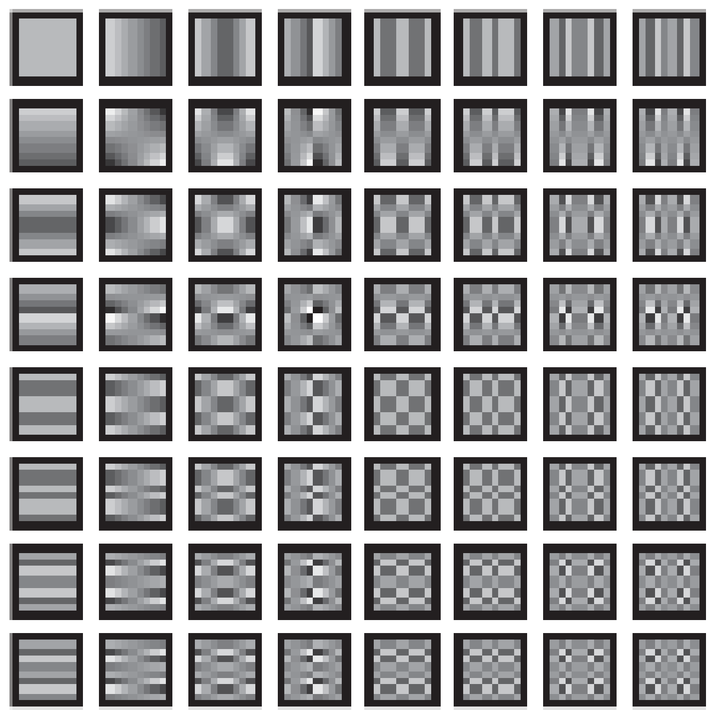
Basis images of matrix-based 2D transforms

Walsh-Hadamard transform (WHT) basis images



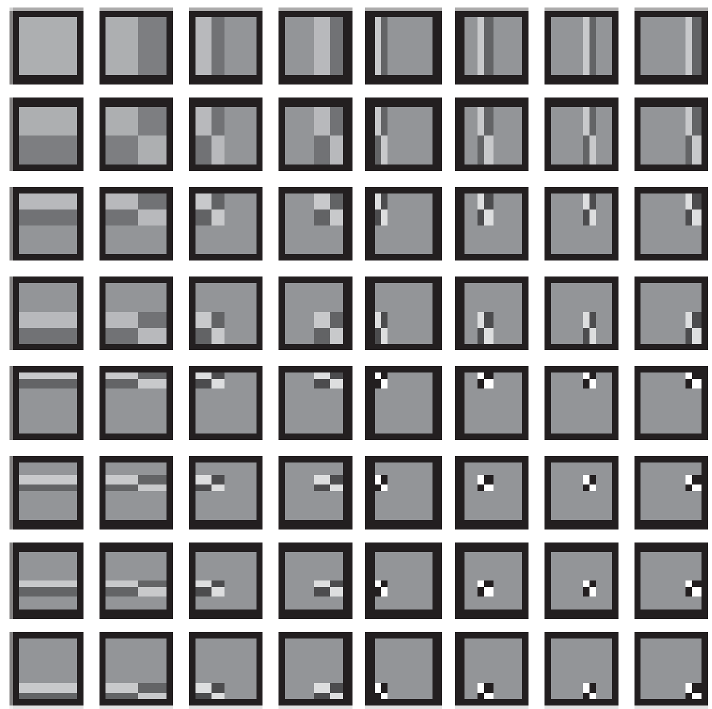
Basis images of matrix-based 2D transforms

Slant transform (SLT) basis images



Basis images of matrix-based 2D transforms

Haar transform (HAAR) basis images



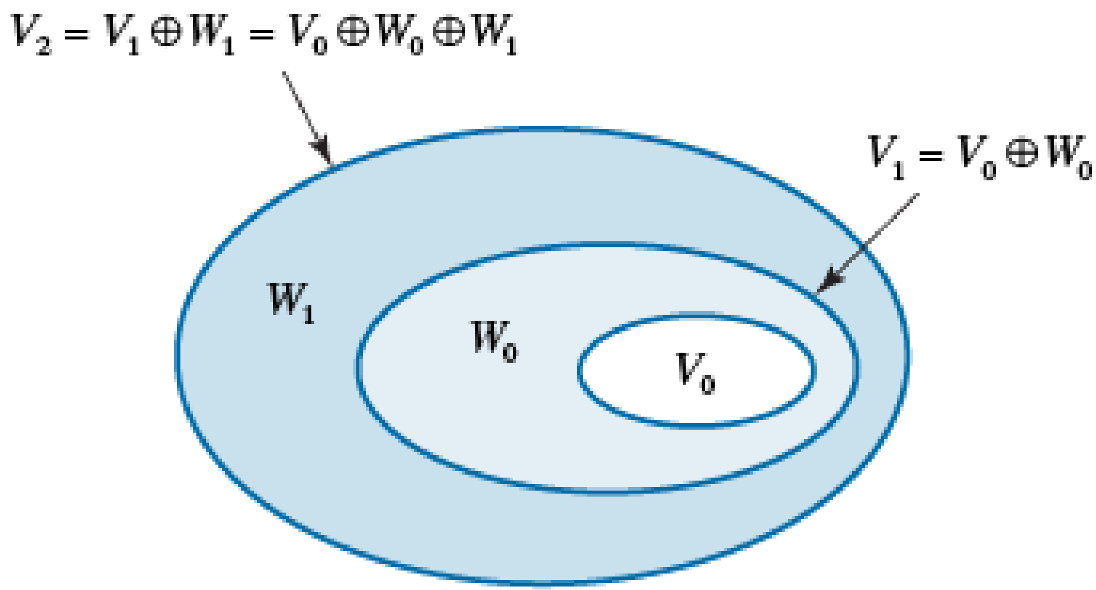
Wavelet transforms

- A scaling function is used to create a series of approximations of a function or image, each differing by a factor of 2 in resolution from its nearest neighboring approximations.
- Wavelet functions (wavelets) are then used to encode the differences between adjacent approximations.
- The discrete wavelet transform (DWT) uses those wavelets, together with a single scaling function, to represent a function or image as a linear combination of the wavelets and scaling function.

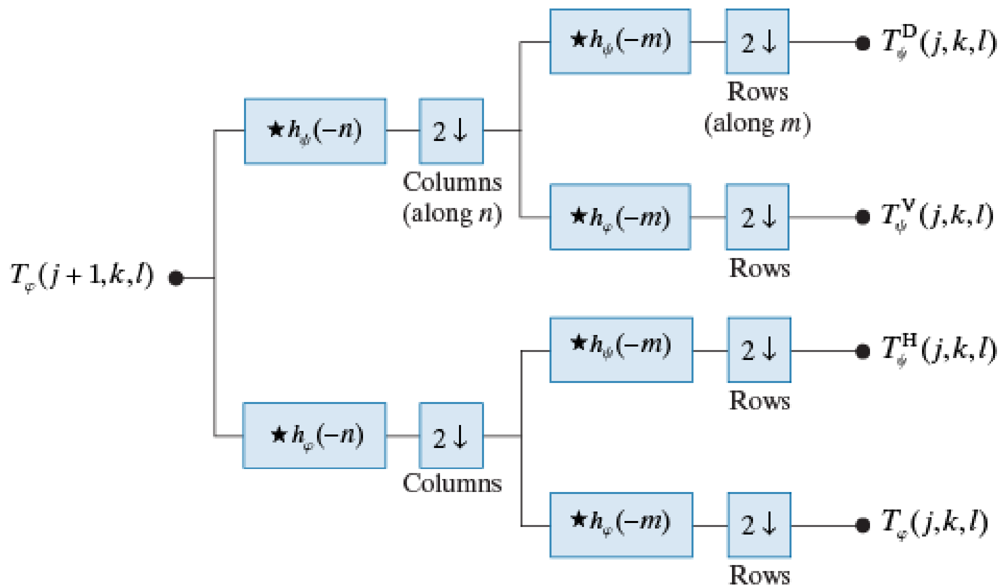
Scaling function, multiresolution analysis

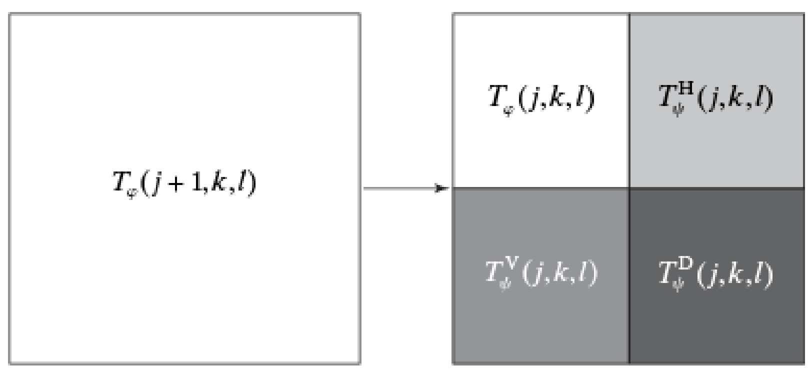
1. The scaling function is orthogonal to its integer translates.
2. The function spaces spanned by the scaling function at low scales are nested within those spanned at higher scales.
   $V_{-\infty} \subset \cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \cdots \subset V_\infty$
3. The only function representable at every scale (all V_j) is $f(x) = 0$.
4. All measureable, square-integrable functions can be represented as $j \to \infty$

Relationship between scaling and wavelet function spaces



2D discrete wavelet transform

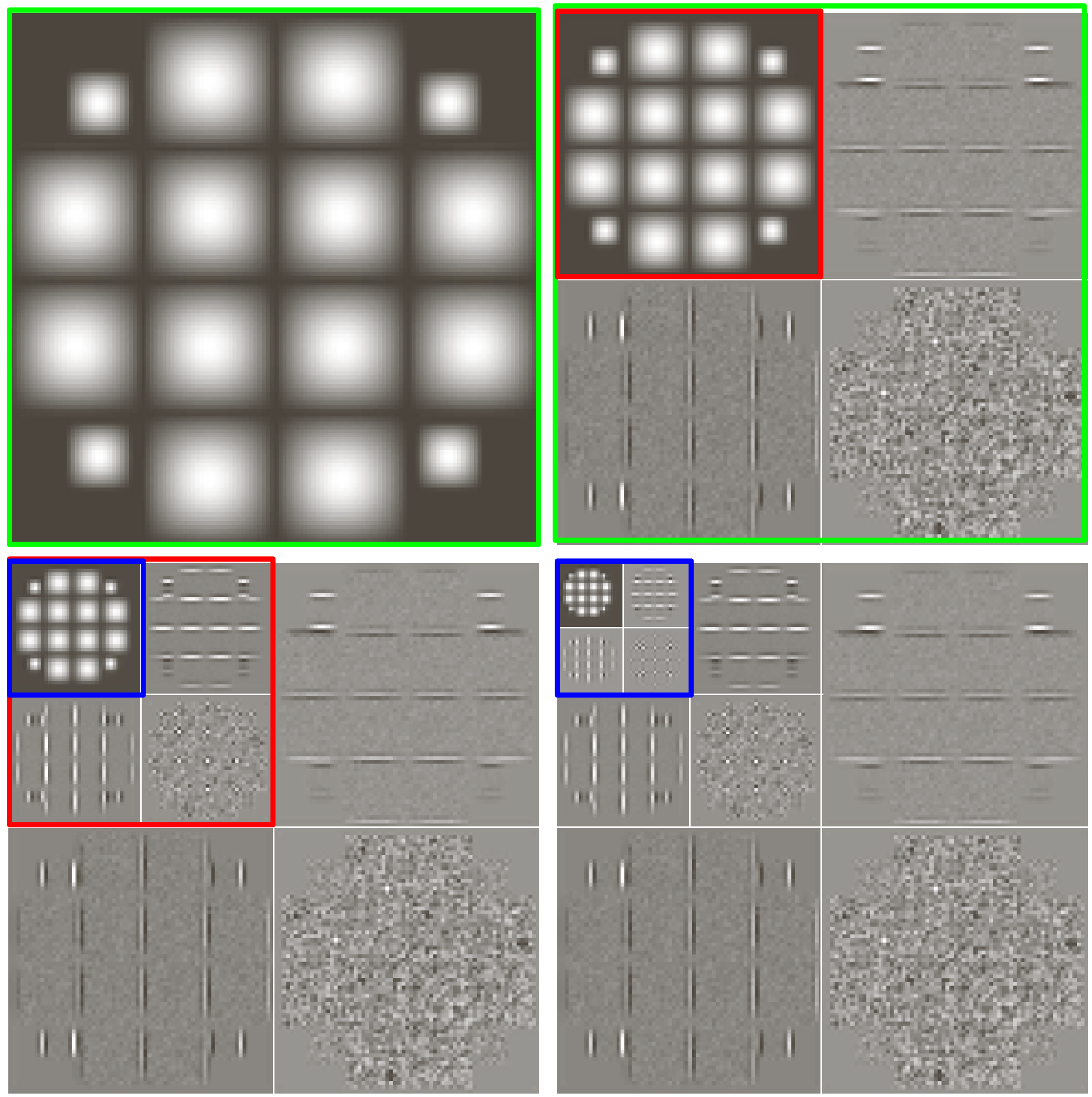




Decomposition

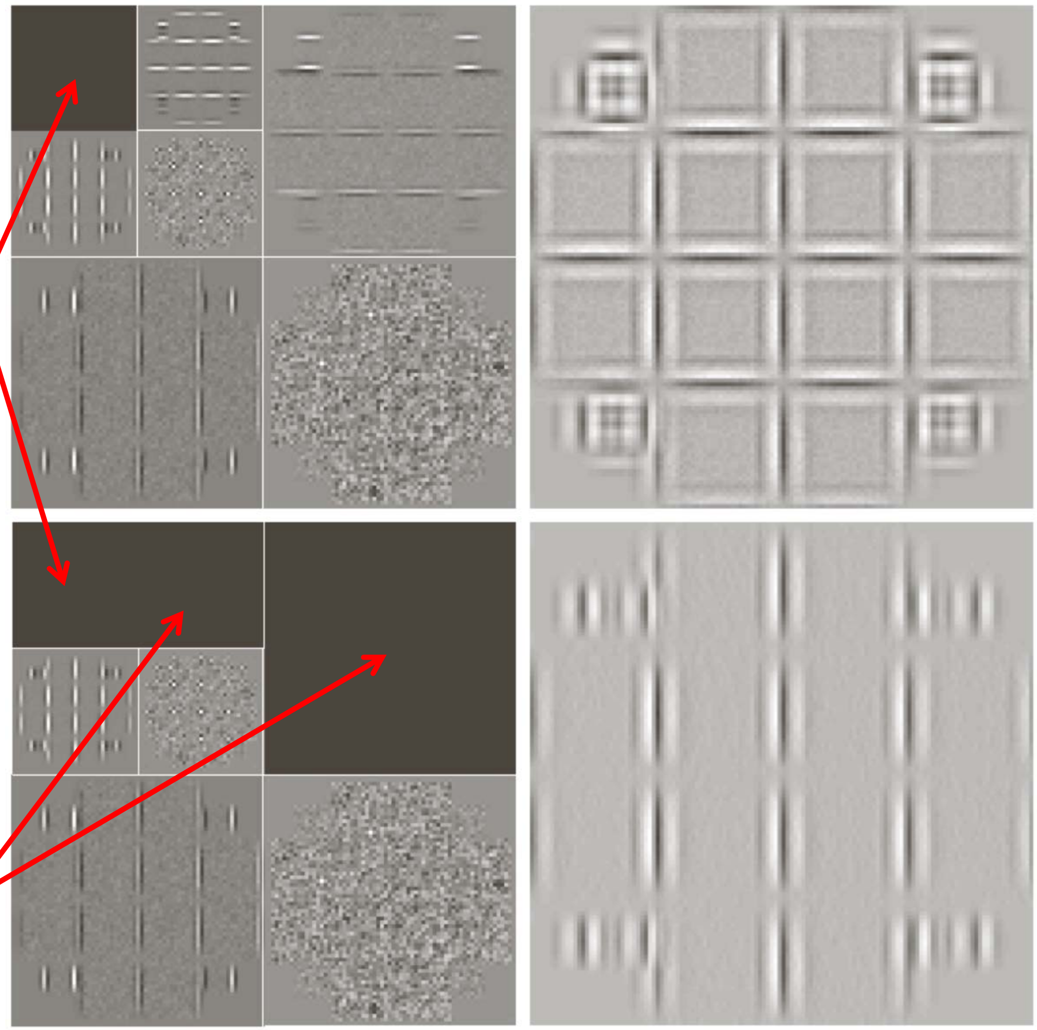
2D discrete wavelet transform

3-level wavelet decomposition



Wavelet-based edge detection

Zero
lowest scale
approximation



Edges

Zero
horizontal
details

Vertical
edges

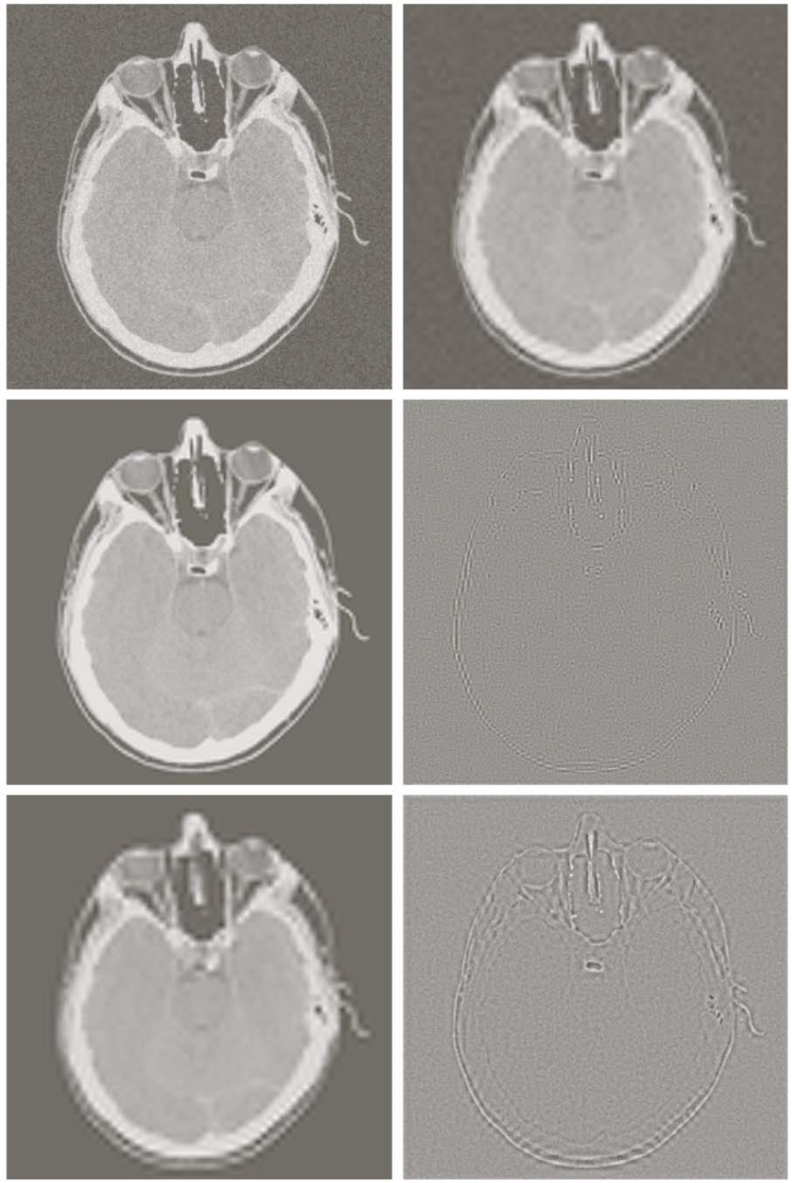
Wavelet-based noise removal

Noisy image

Threshold details

Zero highest resolution details

Zero details for all levels



a	b
c	d
e	f

FIGURE 7.28 Modifying a DWT for noise removal: (a) a noisy CT of a human head; (b), (c) and (e) various reconstructions after thresholding the detail coefficients; (d) and (f) the information removed during the reconstruction of (c) and (e). (Original image courtesy Vanderbilt University Medical Center.)