2D PROJECTIVE TRANSFORMATION, 4 POINT TO STANDARD BASIS OF $\mathbb{P}^2$

STANDARD BASIS OF $\mathbb{P}^2$

$$\underset{\sim}{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \underset{\sim}{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \underset{\sim}{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \underset{\sim}{e}_4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

4 POINTS CAN BE MAPPED TO STANDARD BASIS

$$[\underset{\sim}{e}_1 | \underset{\sim}{e}_2 | \underset{\sim}{e}_3 | \underset{\sim}{e}_4] = \underset{\sim}{H} [s_1 \underset{\sim}{x}_1 | s_2 \underset{\sim}{x}_2 | s_3 \underset{\sim}{x}_3 | s_4 \underset{\sim}{x}_4] \quad \text{DIVIDE BY } s_4$$

$$[\underset{\sim}{e}_1 | \underset{\sim}{e}_2 | \underset{\sim}{e}_3 | \underset{\sim}{e}_4] = \underset{\sim}{H} [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3 | \underset{\sim}{x}_4] \quad \text{WHERE } \lambda_1 = s_1/s_4, \; \lambda_2 = s_2/s_4, \text{ AND } \lambda_3 = s_3/s_4 \; (s_4/s_4 = 1)$$

$$[\underset{\sim}{e}_1 | \underset{\sim}{e}_2 | \underset{\sim}{e}_3] = \underset{\sim}{H} [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

$$\underset{\sim}{I} = \underset{\sim}{H} [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

$$\underset{\sim}{H}^{-1} = [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

$$\underset{\sim}{e}_4 = \underset{\sim}{H} \underset{\sim}{x}_4$$

$$\underset{\sim}{H}^{-1} \underset{\sim}{e}_4 = \underset{\sim}{x}_4$$

$$[\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3] \underset{\sim}{e}_4 = \underset{\sim}{x}_4$$

$$[\underset{\sim}{x}_1 | \underset{\sim}{x}_2 | \underset{\sim}{x}_3] \underset{\sim}{\lambda} = \underset{\sim}{x}_4 \quad \text{WHERE } \underset{\sim}{\lambda} = (\lambda_1, \lambda_2, \lambda_3)^T$$

SOLVE FOR $\underset{\sim}{\lambda}$

$$\underset{\sim}{H}^{-1} = [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \lambda_3 \underset{\sim}{x}_3]$$

NOTE: THIS RESULT SCALES TO n DIMENSIONS