DIRECT METHODS

$I(x,y)$ INTENSITY FUNCTION, REFERENCE IMAGE

$J(x,y)$ INTENSITY FUNCTION

BRIGHTNESS CONSTANCY CONSTRAINT

$$J(x,y) = I(x+u(x,y), y+v(x,y))$$

ASSUME A SMALL DISPLACEMENT

FIRST ORDER TAYLOR EXPANSION

$$J(x,y) = I(x+u(x,y), y+v(x,y)) + uI_x + vI_y + I_t$$

WHERE $I_x = \frac{\partial I}{\partial x}$, $I_y = \frac{\partial I}{\partial y}$, AND $I_t = I - J$

COMBINING PREVIOUS TWO EQUATIONS

$$uI_x + vI_y + I_t = 0$$ DIFFERENTIAL MOTION

$u = t_x$, $v = t_y$

COST FUNCTION $E(t_x, t_y) = \sum [(t_x I_x + t_y I_y + I_t)^2]$

DIRECT - TRANSLATION

TAKING THE PARTIAL DERIVATIVES $\frac{\partial E}{\partial t_x}$ AND $\frac{\partial E}{\partial t_y}$ AND SETTING THIS EXPRESSION TO ZERO

$$\frac{\partial E}{\partial t_x} = \sum [2(t_x I_x + t_y I_y + I_t) I_x] = 0$$

$$2(\sum I_x^2) t_x + 2(\sum I_x I_y) t_y + 2 \sum I_x I_t = 0$$

$$\frac{\partial E}{\partial t_y} = \sum [2(t_x I_x + t_y I_y + I_t) I_y] = 0$$

$$2(\sum I_x I_y) t_x + 2(\sum I_y^2) t_y + 2 \sum I_y I_t = 0$$

IN MATRIX FORM

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y & \sum I_x I_t \\ \sum I_x I_y & \sum I_y^2 & \sum I_y I_t \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix} = 0$$

INHOMOGENEOUS SYSTEM

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$