

# Homework #2

Due: Monday, October 19th, 2015, 10:00 AM

All solutions should be submitted using the `bundleHW2` command on `ieng6`. Include also a collaboration disclosure as done in HW1.

**Problem 1 (DFA design)** For each of the following languages, design a DFA using JFLAP, and submit your solution as a file `HW21a.jff`, `HW21b.jff`, etc. It is highly advisable that you test your automata using JFLAP and/or `haskell` to make sure they are valid DFAs and they recognize the correct language. All languages are over the alphabet  $\{a, b\}$ .

- (a) The set of all strings that begin with “bbab”
- (d) The set of all strings that contain the substring “baa” (the substring should occur as a sequence of contiguous characters)
- (c) The set of all strings that contain an even number of “a”s and an odd number of “b”s.
- (d) The set of all strings that contain between 2 and 4 “b”s.

**Problem 2 (NFA design)** Same as problem 1, but this time you should design an NFA. For full credit, give an NFA with the smallest possible number of states. Submit your solution as files `HW22a.jff`, `HW22b.jff`, etc.

- (a) The set of all strings such that the 4th character is an “a”
- (b) The set of all strings that end with “ba”
- (c) The set of all strings such that the 4th character *from the end* is a “b”.
- (d) The set of all strings that contain the substring “baabb”

**Problem 3 (Closure properties)** Theorem 1.25 in the textbook proves that regular languages are closed under union. The proof is constructive, i.e., it gives an algorithm that on input two DFAs for languages  $L_1$  and  $L_2$ , produces a DFA for  $L_1 \cup L_2$ . In the class notes on Haskell, you have also seen that the proof is immediately translated into a working computer program that implements the transformation. In this problem you are asked to prove and implement some similar closure properties of regular languages.

- (a) Prove that regular languages are closed under intersection by giving a transformation that on input two DFAs for languages  $L_1$  and  $L_2$ , produces a DFA for  $L_1 \cap L_2$ . (Assume all languages are over the same alphabet.)

- (b) For any language  $L \subseteq \Sigma^*$  and symbol  $a \in \Sigma$ , let  $\text{skipLast}(a, L) = \{w \in \Sigma^* \mid wa \in L\}$ , i.e., the set of strings in  $L$  that end in  $a$ , but with that last  $a$  removed. Prove that for any  $L$  and  $a$ , if  $L$  is regular then also  $\text{skipLast}(a, L)$  is regular by giving a transformation that on input a DFA  $M$  with alphabet  $\Sigma$  and a symbol  $a \in \Sigma$ , outputs a DFA for the language  $\text{skipLast}(a, L(M))$ .
- (c) Similar to part (b), but for the language  $\text{skipFirst}(a, L) = \{w \in \Sigma^* \mid aw \in L\}$ , i.e., the set of strings in  $L$  that start in  $a$ , but with that first  $a$  removed.

For each part, your solution should consist of a mathematical description of the transformation and a brief explanation of how/why it works, and a haskell program implementing the transformation. The mathematical proof should be typeset and submitted as a pdf file `HW23x.pdf` (for  $x = a, b, c$ ). For the haskell part, start from the template files `HW23x.hs` provided on the course webpage, and modify them as directed.