Image Formation and Cameras (Part 2)

Computer Vision I

CSE 252A

Lecture 4

Announcements

- Instructor office hours TBD
- TA office hours
  - MW 4:00 PM-5:00 PM
  - New location: EBU3B 4127
- Homework 1 is due Oct 23, 11:59 PM
- Wait list

Planar transformations

- Euclidean (3 degrees of freedom)
  – Rotation and translation
- Similarity (4 degrees of freedom)
  – Rotation, translation, and scale
- Affine (6 degrees of freedom)
  – Linear transformation and translation
- Projective (8 degrees of freedom)
  – Homogeneous 3x3 matrix, defined up to scale

Mapping from a Plane to a Plane under Perspective is given by a projective transform H

x' = Hx H is a 3x3 matrix, x is a 3x1 vector of homogenous coordinates

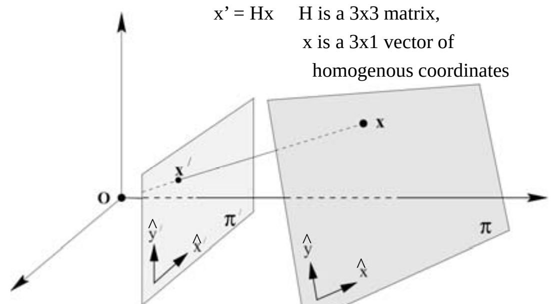


Figure borrowed from Hartley and Zisserman "Multiple View Geometry in computer vision"

Projective transformation

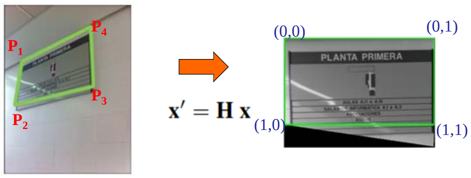
- Also called a homography
- This is a mapping from 2-D to 2-D in homogenous coordinates
- 3 x 3 linear transformation of homogenous coordinates: **u=Ax**

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{21} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- Matrix A is only defined up a scale factor.
- Points map to points
- Lines map to lines

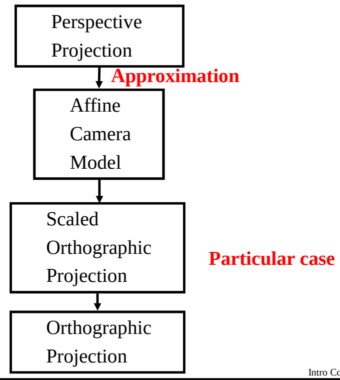
About Homework 1 Replace Add

More applications: OCRs, scan,…

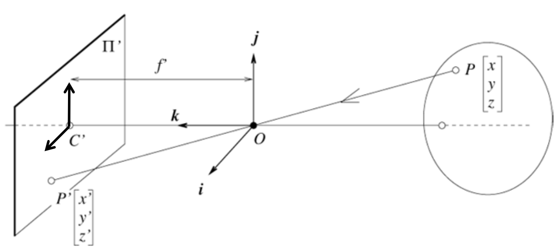


Homography Estimated from four points.

Simplified Camera Models



Equation of Perspective Projection

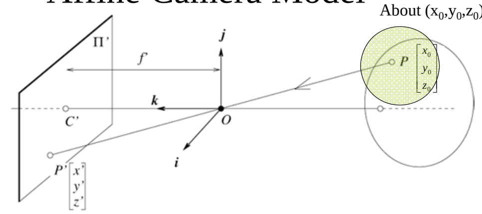


Cartesian coordinates:

- We have, by similar triangles, that (x', y', z') = (f' x/z, f' y/z, f')
- Establishing an image plane coordinate system at C' aligned with i and j, image coordinates of the projection of P are $(x,y,z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$

Affine Camera Model

Appropriate in Neighborhood About (x_0,y_0,z_0)



- Take perspective projection equation, and perform Taylor series expansion about some point P= (x_0, y_0, z_0).
- Drop terms that are higher order than linear.
- Resulting expression is called the affine camera model

- Perspective

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z}\begin{bmatrix} x \\ y \end{bmatrix}$$

- Perform a Taylor series expansion about (x_0, y_0, z_0)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z_0}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \frac{f}{z_0^2}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}(z-z_0) + \frac{f}{z_0}\begin{bmatrix} 1 \\ 0 \end{bmatrix}(x-x_0)$$

$$+\frac{f}{z_0}\begin{bmatrix} 0 \\ 1 \end{bmatrix}(y-y_0) + \frac{f}{2}\frac{2}{z_0^3}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}(z-z_0)^2 + \cdots$$

- Dropping higher order terms and regrouping.

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{f}{z_0}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 \\ 0 & f/z_0 & -fy_0/z_0^2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{Ap}+\mathbf{b}$$

Affine camera model in Euclidean Coordinates

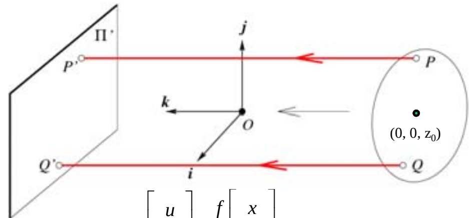
$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{f}{z_0}\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 \\ 0 & f/z_0 & -fy_0/z_0^2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{Ap}+\mathbf{b}$$

Rewrite affine camera model in terms of Homogenous Coordinates

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} f/z_0 & 0 & -fx_0/z_0^2 & fx_0/z_0 \\ 0 & f/z_0 & -fy_0/z_0^2 & fy_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scaled orthographic projection

Starting with Affine Camera Model, take Taylor series about $(x_o, y_0, z_0) = (0, 0, z_0)$ – a point on the optical axis



$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z_0} \begin{bmatrix} x \\ y \end{bmatrix}$$

– That is the z coordinate is dropped, and the image a scaling of the x and y coordinates, where the **scale is $1/z_0$**, the depth of the point of the expansion.

The projection matrix for scaled orthographic projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} f/z_0 & 0 & 0 & 0 \\ 0 & f/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

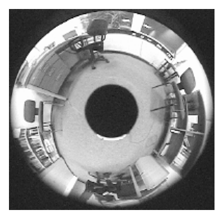
- Parallel lines project to parallel lines
- Ratios of distances are preserved under orthographic projection

For all cameras?

Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnicam (hemispherical)



Light Probe (spherical)

Some Alternative "Cameras"