Image Formation
and Cameras

Computer Vision I

CSE 252A

Lecture 3

Announcements

- http://cseweb.ucsd.edu/classes/fa14/cse252A-b/
- Piazza
- Course reserves available
- Instructor office hours TBD
- Homework 0 is due today by 11:59 PM
- Wait list
- Read:
  - Chapters 1 & 2 of Forsyth & Ponce
  - Chapter 2 of Szeliski (Optional)

Image Formation: Outline

- Factors in producing images
- Projection
- Perspective/Orthographic Projection
- Vanishing points
- Projective Geometry
- Rigid Transformation and SO(3)
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance and Radiometry

Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

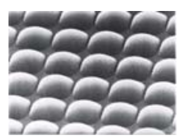
Compare to Paintings

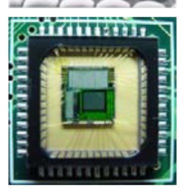
Willem Kalf, Mid 1600’s

Pedro Campos,

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness
- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed





Images are two-dimensional patterns of brightness values.

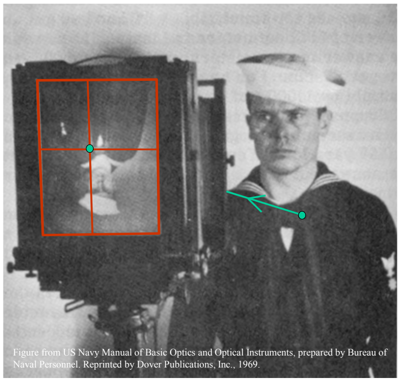


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

They are formed by the projection of 3D objects.

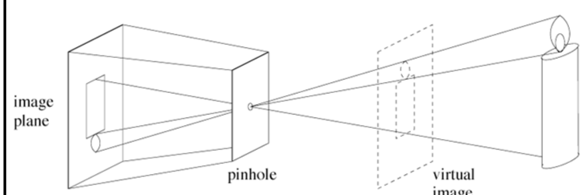
Effect of Lighting: Monet

Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it



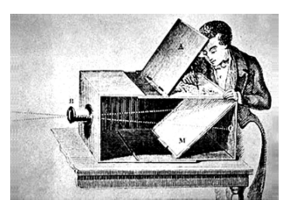
Forsyth&Ponce

Camera Obscura

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". --- Leonardo *Da Vinci*

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

Camera Obscura



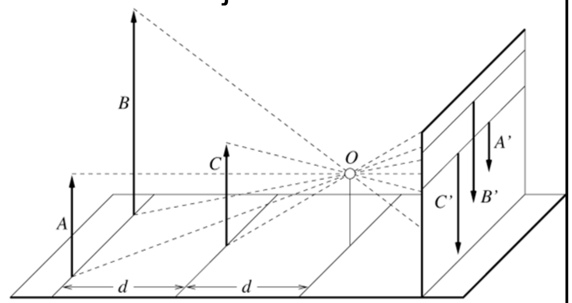
- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

Jetty at Margate England, 1898.

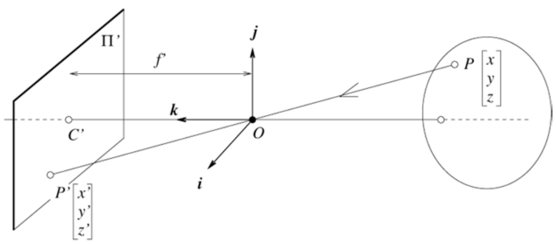
http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

Distant objects are smaller



(Forsyth & Ponce)

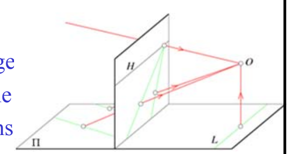
Purely Geometric View of Perspective



The projection of the point **P** on the image plane Π' is given by the point of intersection **P**' of the ray defined by **PO** with the plane Π'.

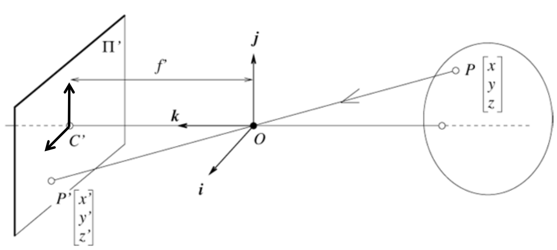
Geometric properties of projection

- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons



- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Degenerate cases:
  - line through focal point project to point
  - plane through focal point projects to a line

Equation of Perspective Projection



Cartesian coordinates:

- We have, by similar triangles, that (x, y, z) -> (f' x/z, f' y/z, f')
- Establishing an image plane coordinate system at C' aligned with i and j, we get $(x,y,z) \to (f\frac{x}{z}, f\frac{y}{z})$

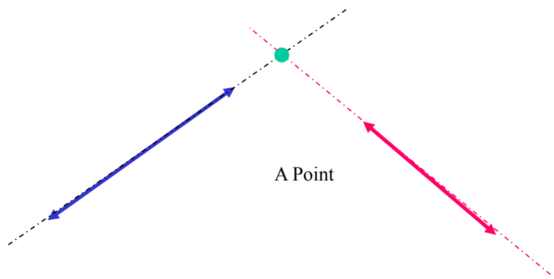
A Digression

Projective Geometry

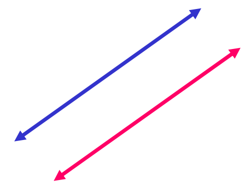
and

Homogenous Coordinates

What is the intersection of two lines in a plane?



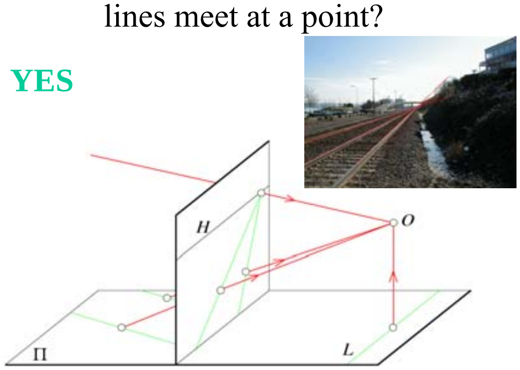
Do two lines in the plane always intersect at a point?



No, Parallel lines don't meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES



Projective geometry provides an elegant means for handling these different situations in a unified way, and **homogenous coordinates** are a way to represent entities (points & lines) in projective spaces.

Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is "bigger" than affine plane – includes "line at infinity"

Homogenous coordinates
A way to represent points in a projective space

- Use three numbers to represent a point on a projective plane

Why? The projective plane has to be bigger than the Cartesian plane.

How: Add an extra coordinate

e.g., (x,y) -> (x,y,1)

Impose equivalence relation

$(x,y,z) \approx \lambda^*(x,y,z)$

such that (λ not 0)

i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$

- Point at infinity – zero for last coordinate e.g., (x,y,0)

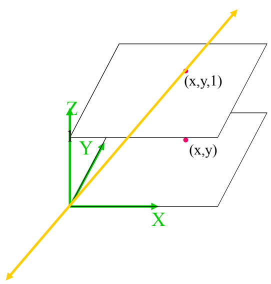
- Why do this?
  - Possible to represent points "at infinity"
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

Homogenous coordinates
A way to represent points in a projective space

Use three numbers to represent a point on a projective plane

Add an extra coordinate
e.g., (x,y) -> (x,y,1)

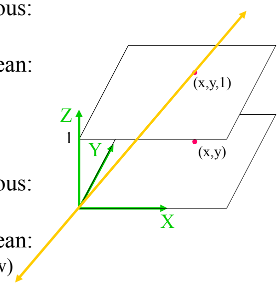
Impose equivalence relation
$(x,y,z) \approx \lambda*(x,y,z)$
such that (λ not 0)
i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$



Conversion
Euclidean -> Homogenous -> Euclidean

In 2-D
- Euclidean -> Homogenous:
  (x, y) -> k (x,y,1)
- Homogenous -> Euclidean:
  (x, y, z) -> (x/z, y/z)

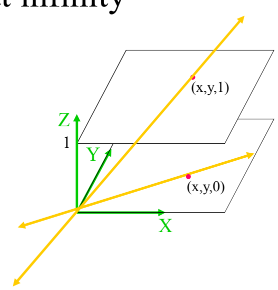
In 3-D
- Euclidean -> Homogenous:
  (x, y, z) -> k (x,y,z,1)
- Homogenous -> Euclidean:
  (x, y, z, w) -> (x/w, y/w, z/w)



Points at infinity

Point at infinity – zero for last coordinate (x,y,0)
and equivalence relation
$(x,y,0) \approx \lambda*(x,y,0)$

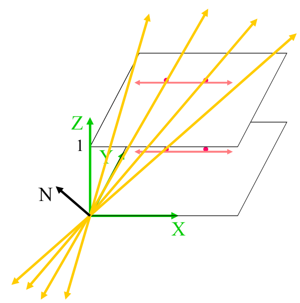
No corresponding Euclidean point



Projective Plane = Affine Plane + Line at Infinity

Lines in Projective space

1. Line in Euclidean plane
2. Plane through origin in homogenous coordinates
3. Plane is represented by its normal **N**
4. Equation for plane is
   N . (x,y,z) = 0
   or **M** . (x,y,z) = 0
   where **M** = λ**N**

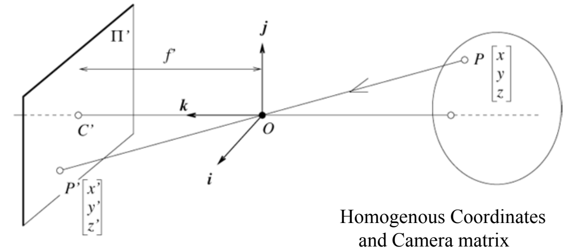


Projective transformation

- 3 x 3 linear transformation of homogenous coordinates
- Points map to points,
- lines map to lines

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{21} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The equation of projection



Cartesian coordinates:

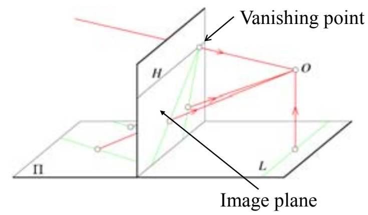
$$(x,y,z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogenous Coordinates and Camera matrix

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

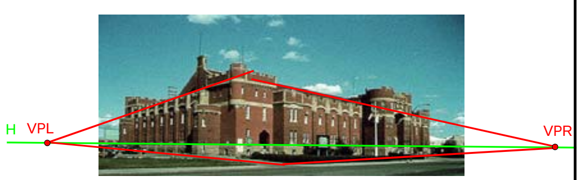
End of the Digression

Parallel lines meet in the image

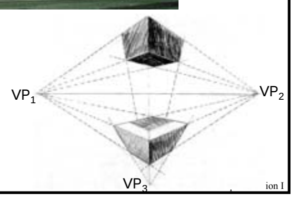


- Formed by line through O
- Parallel to the given line(s)
- A single line can have a vanishing point

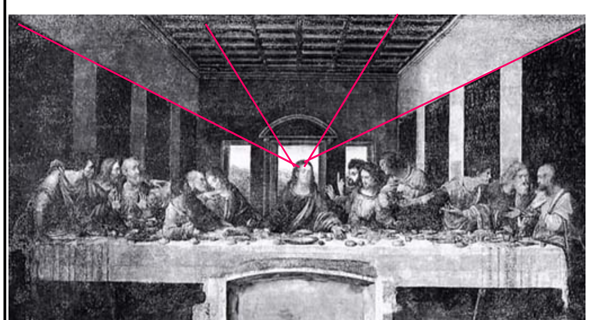
Vanishing points



Different directions correspond different vanishing points



Vanishing Points



Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity.
- The vanishing point is the perspective projection of that point at infinity, resulting from multiplication by the camera matrix.

Projective transformation

- 3 x 3 linear transformation of homogenous coordinates
- Points map to points,
- lines map to lines

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{21} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Mapping from a Plane to a Plane under Perspective is given by a projective transform H

$X' = Hx$

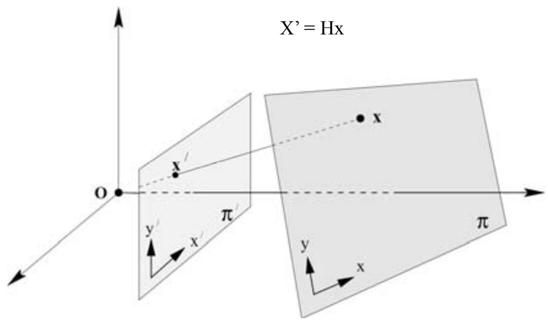
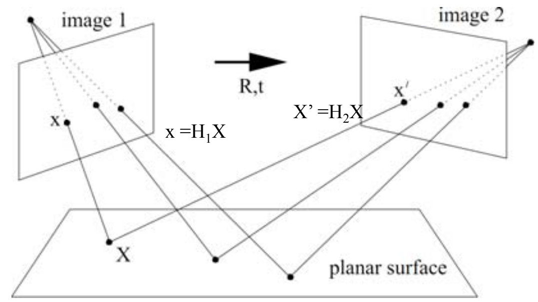


Figure borrowed from Hartley and Zisserman "Multiple View Geometry in computer vision"

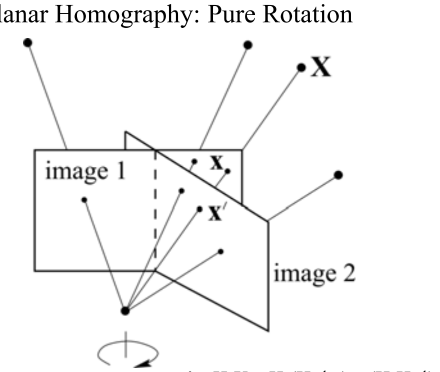
Planar Homography



$x' = H_2X = H_2(H_1^{-1} x) = (H_2H_1^{-1})x$

Figure borrowed from Hartley and Zisserman "Multiple View Geometry in computer vision"

Planar Homography: Pure Rotation



$x' = H_2X = H_2(H_1^{-1} x) = (H_2H_1^{-1})x$

Figure borrowed from Hartley and Zisserman "Multiple View Geometry in computer vision"

Application: Panoramas