Binary Arithmetic and Error Detection

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There are 10 types of people in the world,
those who understand binary,
and those who don’t.

Binary Arithmetic

A Tale of two digits - 1 and 0.

Binary Arithmetic

Four main operations

- Addition
- Subtraction
- Multiplication
- Division

Terminology

Binary Number System

System Digits: 0 and 1

Bit (short for *b*inary dig*it*): A single binary digit

LSB (least significant bit): The rightmost bit

MSB (most significant bit): The leftmost bit

Binary Equivalents

1 Nibble = 4 bits

1 Byte = 2 nibbles = 8 bits

1. Binary Addition

Eg: 00010011 + 00111110 = 01010001

Rules :

0 + 0 = 0
0 + 1 = 1
1 + 0 = 1
1 + 1 = 0

		1	*1*	*1*	*1*	*1*			*carries*
	0	0	0	1	0	0	1	1	= $19_{(\text{base } 10)}$
+	0	0	1	1	1	1	1	0	= $62_{(\text{base } 10)}$
	0	1	0	1	0	0	0	1	= $81_{(\text{base } 10)}$

2. Binary Subtraction

Eg: 00110011 - 00010110 = 00011101

Rules :

0 - 0 = 0
0 - 1 = 1
1 - 0 = 1
1 - 1 = 0

		0	*1 0*	*1*					*borrows*
	0	0	~~1~~	~~1~~	~~0~~	1 0	1	1	= 51 (base 10)
−	0	0	0	1	0	1	1	0	= 22 (base 10)
	0	0	0	1	1	1	0	1	= 29 (base 10)

3. Binary Multiplication

Rules :

0 x 0 = 0
0 x 1 = 0
1 x 0 = 0
1 x 1 = 1

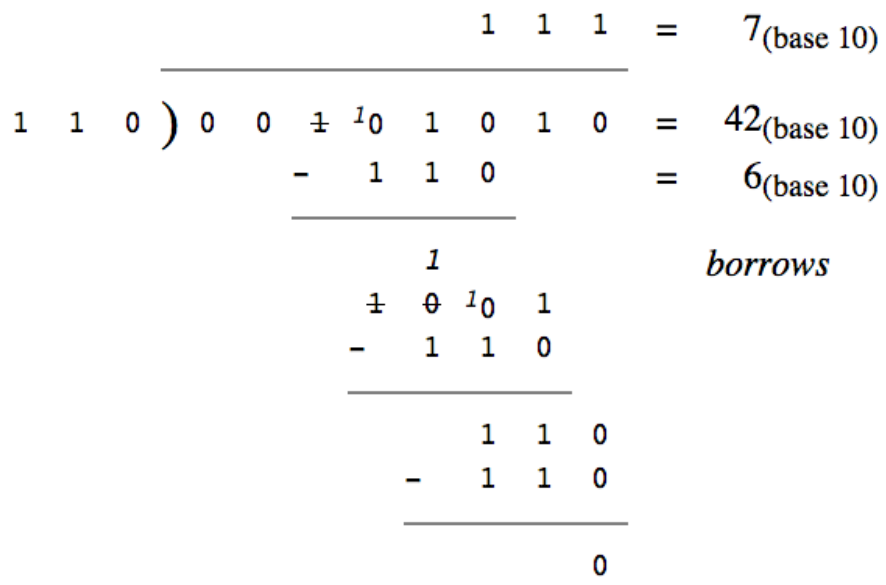
Eg: 00101001 × 00000110 = 11110110

		0	0	1	0	1	0	0	1	= $41_{(\text{base } 10)}$
	×	0	0	0	0	0	1	1	0	= $6_{(\text{base } 10)}$
		0	0	0	0	0	0	0	0	
	0	0	1	0	1	0	0	1		
0	0	1	0	1	0	0	1			
0	0	1	1	1	1	0	1	1	0	= $246_{(\text{base } 10)}$

4. Binary Division

Eg: 00101010 ÷ 00000110 = 00000111

Rules :
Repeated subtraction.



Bitwise Operations

Four main bitwise operations :

1. NOT
2. AND
3. OR
4. XOR

XOR Rules :

$0 \oplus 0 = 0$

$0 \oplus 1 = 1$

$1 \oplus 0 = 1$

$1 \oplus 1 = 0$

Error Detection

1. Two Dimensional Parity

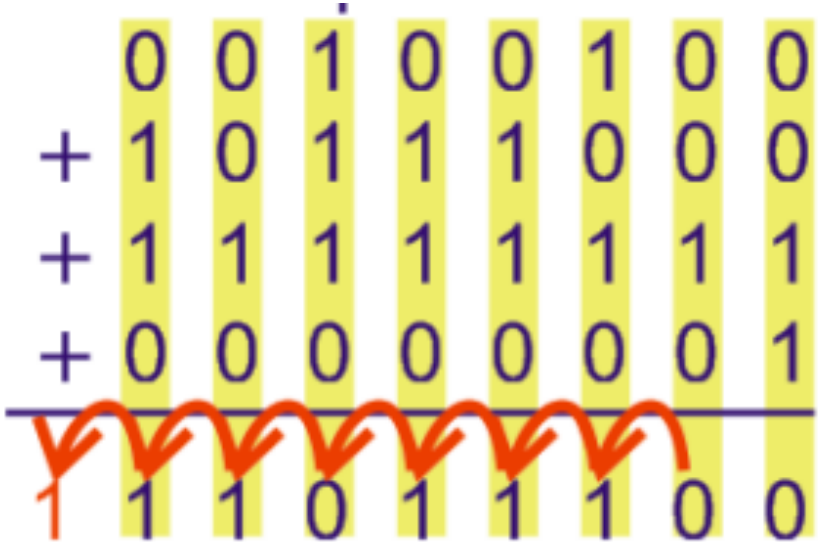
Data
1100
1011
0111
0101

				row parity
1	1	0	0	0
1	0	1	1	1
0	1	1	1	1
0	1	0	1	0
0	1	0	1	0 (matrix parity bit)

col parity bits

2. Checksum

Input : 00100100 , 10111000, 11111111, 00000001



3. CRC - Cyclic Redundancy Check

Dividend - 11010011101100

Generator polynomial - x^3+x+1

References :

- http://academic.evergreen.edu/
- http://www.cs.utexas.edu/~eberlein/cs337/errorDetection3.pdf

Thank You!