State Minimization of Completely Specified FSMs

Equivalent States

- Two states A and B are “equivalent” if the output sequence is the same when given the same input sequence

- We can “merge” equivalent states together to minimize number of states

Implication Chart

- Shows if a pair of state is still “compatible”
- Not compatible if outputs are different
- Not compatible if corresponding next states are “no longer compatible”
- Iterate until no more pairs of states can be “crossed out”

Example and 1st Iteration

PS	Next State		Out
	X=0	X=1	
A	B	C	0
B	A	C	0
C	D	C	0
D	D	E	1
E	A	F	0
F	B	G	0
G	A	E	0

Cross out "squares" where outputs conflict

	A	B	C	D	E	F
B	A-B C-C					
C	B-D C-C	A-D C-C				
D						
E	A-B C-F	A-A C-F	A-D C-F			
F	B-B C-G	A-B C-G	B-D C-G		A-B F-G	
G	A-B C-E	A-A C-E	A-D C-E		A-A E-F	A-B E-G

2nd Iteration

	A	B	C	D	E	F
B	A-B C-C					
C	~~B-D C-C~~	~~A-D C-C~~				
D						
E	A-B C-F	A-A C-F	~~A-D C-F~~			
F	B-B C-G	A-B C-G	~~B-D C-G~~		A-B F-G	
G	A-B C-E	A-A C-E	~~A-D C-E~~		A-A E-F	A-B E-G

3rd Iteration

	A	B	C	D	E	F
B	A-B C-C					
C	~~B-D~~ ~~C-C~~	~~A-D~~ ~~C-C~~				
D						
E	~~A-B~~ ~~C-F~~	~~A-A~~ ~~C-F~~	~~A-D~~ ~~C-F~~			
F	~~B-B~~ ~~C-G~~	~~A-B~~ ~~C-G~~	~~B-D~~ ~~C-G~~		A-B F-G	
G	~~A-B~~ ~~C-E~~	~~A-A~~ ~~C-E~~	~~A-D~~ ~~C-E~~		A-A E-F	A-B E-G

Final:
A = B
E = F = G

Note:
If E = F, F = G, then E = G must also be true

Reduced State Table

PS	Next State		Out
	X=0	X=1	
A	A	C	0
C	D	C	0
D	D	E	1
E	A	E	0

Example 2

PS	Next State		Output	
	X=0	X=1	X=0	X=1
A	B	E	0	0
B	C	B	0	0
C	B	G	0	0
D	B	D	0	0
E	F	E	0	0
F	C	B	0	0
G	F	D	0	1

1st Iteration

	A	B	C	D	E	F
B	B-C E-B					
C	B-B E-G	C-B B-G				
D	B-B E-D	C-B B-D	B-B G-D			
E	B-F E-E	C-F B-E	B-F G-E	B-F D-E		
F	B-C E-B	C-C B-B	B-C G-B	B-C D-B	F-C E-B	
G						

Cross out "squares" where outputs conflict

2nd Iteration

	A	B	C	D	E	F
B	B-C E-B					
C	~~B-B E-G~~	~~C-B B-G~~				
D	B-B E-D	C-B B-D	~~B-B G-D~~			
E	B-F E-E	C-F B-E	~~B-F G-E~~	B-F D-E		
F	B-C E-B	C-C B-B	~~B-C G-B~~	B-C D-B	F-C E-B	
G						

3rd Iteration

	A	B	C	D	E	F
B	~~B-C E-B~~					
C	~~B-B E-G~~	~~C-B B-G~~				
D	B-B E-D	~~C-B B-D~~	~~B-B G-D~~			
E	B-F E-E	~~C-F B-E~~	~~B-F G-E~~	B-F D-E		
F	~~B-C E-B~~	C-C B-B	~~B-C G-B~~	~~B-C D-B~~	~~F-C E-B~~	
G						

Final:
A = D = E
B = F

Reduced State Table

PS	Next State		Output	
	X=0	X=1	X=0	X=1
A	B	A	0	0
B	C	B	0	0
C	B	G	0	0
G	F	D	0	1

Example 3

PS	Next State		Out
	X=0	X=1	
A	A	C	0
B	D	B	1
C	D	B	0
D	B	C	0
E	E	F	0
F	G	B	0
G	B	F	0

1st Iteration

	A	B	C	D	E	F
B						
C	A-D C-B					
D	A-B C-C		D-B B-C			
E	A-E C-F		D-E B-F	B-E C-F		
F	A-G C-B		D-G B-B	B-G C-B	E-G F-B	
G	A-B C-F		D-B B-F	B-B C-F	E-B F-F	G-B B-F

Cross out "squares" where outputs conflict

2nd Iteration

	A	B	C	D	E	F
B						
C	A-D C-B					
D	A-B C-C		D-B B-C			
E	A-E C-F		D-E B-F	B-E C-F		
F	A-G C-B		D-G B-B	B-G C-B	E-G F-B	
G	A-B C-F		D-B B-F	B-B C-F	E-B F-F	G-B B-F

Final:
A = E
C = F
D = G

Reduced State Table

PS	Next State		Out
	X=0	X=1	
A	A	C	0
B	D	B	1
C	D	B	0
D	B	C	0

Example 4

PS	Next State		Out
	X=0	X=1	
A	F	C	0
B	A	B	1
C	A	F	1
D	D	B	1
E	A	F	1
F	A	F	1
G	F	E	0

1st Iteration

	A	B	C	D	E	F
B						
C		A-A B-F				
D		A-D B-B	A-D F-B			
E		A-A B-F	A-A F-F	D-A B-F		
F		A-A B-F	A-A F-F	D-A B-F	A-A F-F	
G	F-F C-E					

Cross out "squares" where outputs conflict

2nd Iteration

	A	B	C	D	E	F
B						
C		A-A B-F				
D		~~A-D B-B~~	~~A-D F-B~~			
E		A-A B-F	A-A F-F	~~D-A B-F~~		
F		A-A B-F	A-A F-F	~~D-A B-F~~	A-A F-F	
G	F-F C-E					

Final:
A = G
B = C = E = F

Reduced State Table

PS	Next State		Out
	X=0	X=1	
A	B	B	0
B	A	B	1
D	D	B	1