

1) Number representation

a) Given X and Y below that are 4-bit numbers in 2's complement form, fill in the table of what S should be bit-vector form, whether or not there is an *overflow*, and what S is as a number. (First row provides an example.)

X $x_3x_2x_1x_0$	Y $y_3y_2y_1y_0$	$S = X - Y$ $s_3s_2s_1s_0$	Overflow?	S as a number
0111 (7)	0011 (3)	0100	No	4
1000 (-8)	0010 (2)	x	Yes	-10(x)
1111 (-1)	1010 (-6)	0101	No	5
0110 (6)	1011 (-5)	x	Yes	11(x)
0000 (0)	0001 (1)	1111	No	-1
0101 (5)	0111 (7)	1110	No	-2

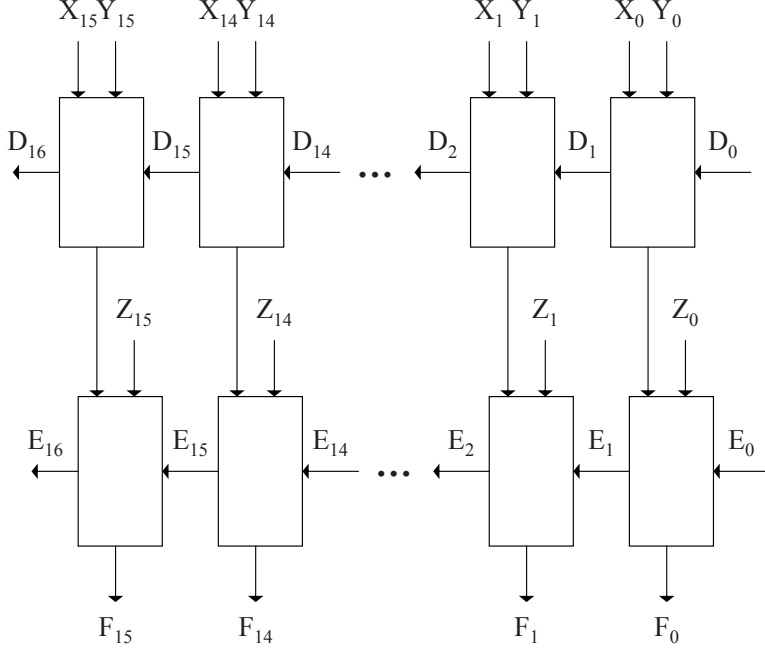
“x” means number is not valid (because an overflow has occurred).

b) For each binary vector below, what does it represent as an Octal number and a Hexadecimal number?

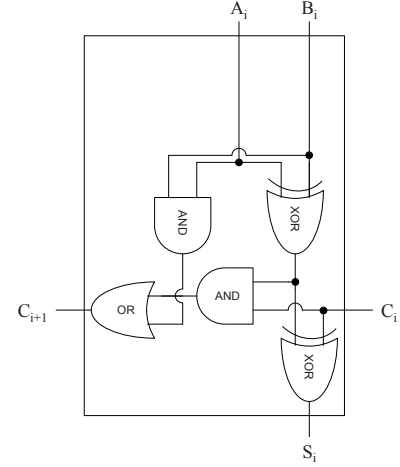
Binary	Octal	Hexadecimal
000111	07	07
101001	51	29
100100	44	24

2) Delay analysis

Consider the following 16-bit adder design that adds the numbers $F = X + Y + Z$. Assume **1ns** gate delay for all gates (i.e., 2-XOR, 2-AND, and 2-OR).



The logic for each bitslice (rectangle box) is as follows:



What are final delays for the following:

$$F_0 = 4$$

$$F_1 = 6$$

$$F_{14} = 32$$

$$F_{15} = 34$$

$$D_1 = 3$$

$$D_{14} = 29$$

$$D_{15} = 31$$

$$E_1 = 5$$

$$E_{14} = 31$$

$$E_{15} = 33$$

More generally,

$$F_i = E_i + 1, \forall i > 0$$

$$D_{i+1} = D_i + 2, \forall i > 1$$

$$E_{i+1} = E_i + 2, \forall i > 0$$

3) Two-level logic minimization

$$F(A, B, C, D) = \sum m(3, 6, 8, 9, 15)$$

$$d(A, B, C, D) = \sum m(1, 7, 13)$$

		CD			
		00	01	11	10
AB	00		x	1	
	01			x	1
	11		x	1	
	10	1	1		

a) Identify all the prime and essential prime implicants.

	Prime	Essential
100-	$A\bar{B}\bar{C}$	X
11-1	ABD	
011-	$\bar{A}BC$	X
00-1	$\bar{A}\bar{B}D$	
1-01	$A\bar{C}D$	
-111	BCD	
0-11	$\bar{A}CD$	
-001	$\bar{B}\bar{C}D$	

b) Find the minimum two-level logic implementation.

$$\begin{aligned}
 F &= A\bar{B}\bar{C} + \bar{A}BC + ABD + \bar{A}\bar{B}D \text{ OR} \\
 &= A\bar{B}\bar{C} + \bar{A}BC + ABD + \bar{A}CD \text{ OR} \\
 &= A\bar{B}\bar{C} + \bar{A}BC + BCD + \bar{A}CD \text{ OR} \\
 &= A\bar{B}\bar{C} + \bar{A}BC + BCD + \bar{A}\bar{B}D
 \end{aligned}$$

3) Multi-level logic minimization

Given following logic equations, minimize the number of literals, e.g. by using common subexpressions, Boolean rules, etc. You can introduce new immediate equations for common sub-expressions if it helps to reduce the number of literals. Put a box around your final answer, and indicate the number of literals in your final answer.

$$\begin{aligned}H &= (A\bar{B} + B + C)(B + 1) \\&= (A + B + C)(1) \\&= A + B + C\end{aligned}$$

$$\begin{aligned}J &= \bar{A}\bar{B}\bar{C}E + \bar{A}\bar{B}\bar{C}\bar{E} + \bar{A}\bar{B}\bar{C}F \\&= \bar{A}\bar{B}\bar{C}(E + \bar{E} + F) \\&= \bar{A}\bar{B}\bar{C}(1) \\&= \bar{A}\bar{B}\bar{C} \\&= \bar{H}\end{aligned}$$

$$\begin{aligned}K &= AE + BE + CE + AF + BF + CF \\&= (A + B + C)(E + F) \\&= H(E + F)\end{aligned}$$

$$\begin{aligned}L &= \bar{A}(B + C) + A \\&= (B + C) + A \\&= H\end{aligned}$$

Final answer (8 literals):

$$\begin{aligned}H &= A + B + C \\J &= \bar{H} \\K &= H(E + F) \\L &= H\end{aligned}$$