

1) Number representation

a) For each binary vector below, what does it represent as an Octal number and a Hexadecimal number?

Binary	Octal	Hexadecimal
10110011	263	B3
01111100	174	7C
11111111	377	FF

b) Given X and Y below that are 5-bit numbers in 2's complement form, fill in the table of what S should be bit-vector form (**even if the result causes an overflow**). Indicate whether or not there is an *overflow* and what S is as a number (if S results in an overflow, then just put an "x" under the S column). (First two rows provide examples.)

X $x_4x_3x_2x_1x_0$	Y $y_4y_3y_2y_1y_0$	$S = X - Y$ $s_4s_3s_2s_1s_0$	Overflow?	S as a number
11101	00011	11010	no	-6
00001	00111	11010	no	-6
01000	10010	10110	yes	x
10111	10111	00000	no	0
10111	01011	01100	yes	x
01100	00101	00111	no	7
10101	10111	11110	no	-2

2) Multi-level logic minimization

Given following logic equations, minimize the number of literals, e.g. by using common subexpressions, Boolean rules, etc. You can introduce new immediate equations for common sub-expressions if it helps to reduce the number of literals. Put a box around your final answer, and indicate the number of literals in your final answer.

$$\begin{aligned}
 S &= \bar{A}\bar{B}C \\
 P &= \overline{(A+B+\bar{C}+\bar{D})(A+B+\bar{C}+\bar{E})} + \overline{(A+B+\bar{C}+\bar{A}\bar{D})} \\
 &= \bar{A}\bar{B}CD + \bar{A}\bar{B}CE + \bar{A}\bar{B}C(\bar{A}\bar{D}) \\
 &= \bar{A}\bar{B}CD + \bar{A}\bar{B}CE + \bar{A}\bar{B}C(A+D) \\
 &= \bar{A}\bar{B}CD + \bar{A}\bar{B}CE + \bar{A}\bar{B}C(A+D) \\
 &= \bar{A}\bar{B}CD + \bar{A}\bar{B}CE + \bar{A}\bar{B}CD \\
 &= \bar{A}\bar{B}C(D+E) \\
 &= S(D+E) \\
 Q &= \bar{A}\bar{B}CF + \bar{A}\bar{B}CG + \bar{A}\bar{B}CH + XYF + XYG + XYH \\
 &= \bar{A}\bar{B}C(F+G+H) + XY(F+G+H) \\
 &= (F+G+H)(\bar{A}\bar{B}C + XY) \\
 &= (F+G+H)(S + XY) \\
 R &= F + G + H + \bar{F}\bar{G} \\
 &= 1
 \end{aligned}$$

Final answer (12 literals):

$$\begin{aligned}
 S &= \bar{A}\bar{B}C \\
 P &= S(D+E) \\
 Q &= (F+G+H)(S+XY) \\
 R &= 1
 \end{aligned}$$

3) Two-level logic minimization

$$F(A, B, C, D) = \sum m(2, 3, 5, 7, 8, 10, 15)$$

$$d(A, B, C, D) = \sum m(1, 12, 13)$$

		CD			
		00	01	11	10
AB	00		x	1	1
	01		1	1	
	11	x	x	1	
	10	1			1

a) Identify all the prime and essential prime implicants.

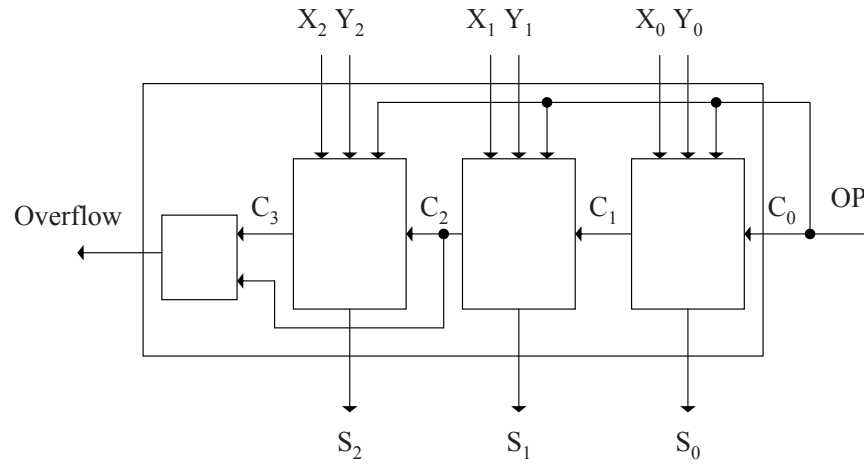
	Prime	Essential
0-1	$\bar{A}D$	X
001-	$\bar{A}\bar{B}C$	
-1-1	BD	
110-	$AB\bar{C}$	
1-00	$A\bar{C}\bar{D}$	
10-0	$A\bar{B}\bar{D}$	
-010	$\bar{B}C\bar{D}$	

b) Find the minimum two-level logic implementation.

$$F = \bar{A}\bar{B}C + BD + A\bar{B}\bar{D}$$

4) Multiplexor/Adder/Subtractor

Implement a 3-bit adder/subtractor using only 2:1 multiplexors and inverters. When $OP = 0$, do an addition. Else (when $OP = 1$), do a subtraction.



The equations are:

$$\begin{aligned}
 \text{Overflow} &= C_3 \oplus C_2 & B_i &= X_i \oplus Z_i \\
 C_0 &= OP & S_i &= B_i \oplus C_i \\
 Z_i &= OP \oplus Y_i & C_{i+1} &= X_i Z_i + C_i B_i
 \end{aligned}$$

