

1) Number representation

Given X and Y below that are 5-bit numbers in 2's complement form, fill in the table of what S should be as a 5-bit number in 2's complement form (**even if the result causes an overflow**). Indicate whether or not there is an *overflow* and what S is as a number (if S results in an overflow, then just put an "x" under the S column).

Also, in the last two columns, specify how the binary vector of X should be interpreted as an Octal number and a Hexadecimal number, respectively?

(First two rows provide examples.)

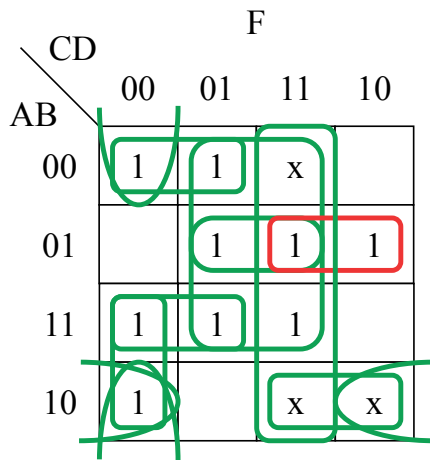
X $x_4x_3x_2x_1x_0$	Y $y_4y_3y_2y_1y_0$	$S = X + Y$ $s_4s_3s_2s_1s_0$	Overflow?	S in 2's complement	X in Octal	X in Hexadecimal
11101	10001	01110	yes	x	35	1D
00001	00111	01000	no	8	01	01
00111	01001	10000	yes	x	07	07
00111	10111	11110	no	-2	07	07
11001	10111	10000	no	-16	31	19
11001	01001	00010	no	2	31	19

Note: $X = 11001$ is the same for the last two rows. Hence X in Octal should be the same for the last two rows as well. Same with X in Hexadecimal.

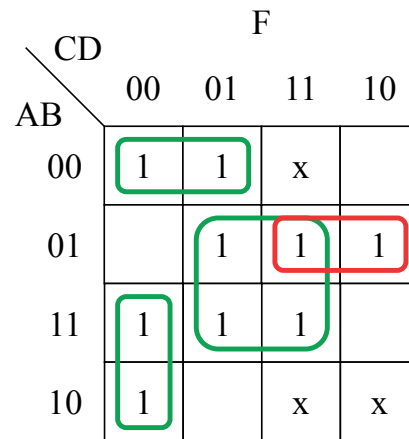
2) Two-level logic minimization

$$F(A, B, C, D) = \sum m(0, 1, 5, 6, 7, 8, 12, 13, 15)$$

$$d(A, B, C, D) = \sum m(3, 10, 11)$$



All Primes



Minimal Solution

a) Identify all the prime and essential prime implicants.

	Prime	Essential
— — 11	CD	$\bar{A}BC$
0 — —1	$\bar{A}D$	
—1 — 1	BD	
000—	$\bar{A}\bar{B}\bar{C}$	
011—	$\bar{A}BC$	
110—	$AB\bar{C}$	
10 — 0	$A\bar{B}\bar{D}$	
101—	$A\bar{B}C$	
—000	$\bar{B}\bar{C}\bar{D}$	
1 — 00	$A\bar{C}\bar{D}$	

b) Find the minimum two-level logic implementation.

$$F = \bar{A}BC + BD + A\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}$$

3) Multi-level logic minimization

Given following logic equations, minimize the number of literals, e.g. by using common subexpressions, Boolean rules, etc. You can introduce new immediate equations for common sub-expressions if it helps to reduce the number of literals. Put a box around your final answer, and indicate the number of literals in your final answer.

$$\begin{aligned}
X &= \bar{A}B + C + DE \\
Y &= FG \\
M &= A\bar{C}\bar{D}FG + \bar{B}\bar{C}\bar{D}FG + A\bar{C}\bar{E}FG + \bar{B}\bar{C}\bar{E}FG + \bar{A}B\bar{F} + \bar{A}B\bar{G} + C\bar{F} + C\bar{G} + DE\bar{F} + DE\bar{G} \\
&= (A\bar{D} + \bar{B}\bar{D} + A\bar{E} + \bar{B}\bar{E})\bar{C}FG + (\bar{A}B + C + DE)(\bar{F} + \bar{G}) \\
&= (A + \bar{B})\bar{C}(\bar{D} + \bar{E})FG + (\bar{A}B + C + DE)(\bar{F} + \bar{G}) \\
&= \bar{X}Y + X\bar{Y} \\
N &= ABC + FG + \bar{A}F + \bar{B}F + \bar{C}F \\
&= ABC + FG + (\bar{A} + \bar{B} + \bar{C})F \\
&= ABC + FG + F \\
&= ABC + F \\
P &= \bar{A}BF + CF + DEF + \bar{A}B\bar{F} + C\bar{F} + DE\bar{F} + \bar{C}H + FH \\
&= (\bar{A}B + C + DE)(F + \bar{F}) + (\bar{C} + F)H \\
&= (\bar{A}B + C + DE) + H \\
&= X + H
\end{aligned}$$

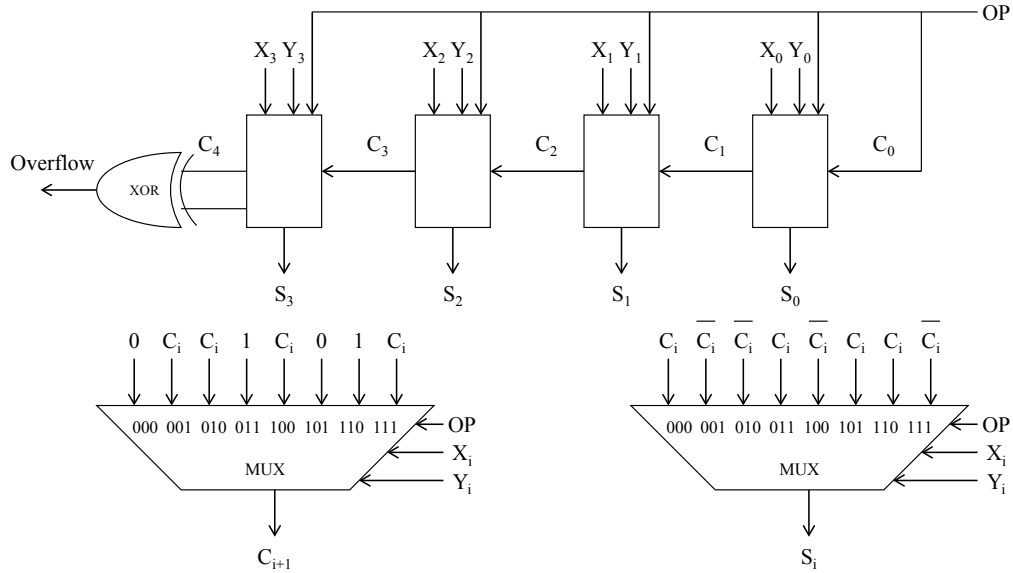
OR no separate Y equation.

$$\begin{aligned}
X &= \bar{A}B + C + DE \\
M &= A\bar{C}\bar{D}FG + \bar{B}\bar{C}\bar{D}FG + A\bar{C}\bar{E}FG + \bar{B}\bar{C}\bar{E}FG + \bar{A}B\bar{F} + \bar{A}B\bar{G} + C\bar{F} + C\bar{G} + DE\bar{F} + DE\bar{G} \\
&= (A\bar{D} + \bar{B}\bar{D} + A\bar{E} + \bar{B}\bar{E})\bar{C}FG + (\bar{A}B + C + DE)(\bar{F} + \bar{G}) \\
&= (A + \bar{B})\bar{C}(\bar{D} + \bar{E})FG + (\bar{A}B + C + DE)(\bar{F} + \bar{G}) \\
&= \bar{X}FG + X(\bar{F} + \bar{G}) \\
N &= ABC + FG + \bar{A}F + \bar{B}F + \bar{C}F \\
&= ABC + FG + (\bar{A} + \bar{B} + \bar{C})F \\
&= ABC + FG + F \\
&= ABC + F \\
P &= \bar{A}BF + CF + DEF + \bar{A}B\bar{F} + C\bar{F} + DE\bar{F} + \bar{C}H + FH \\
&= (\bar{A}B + C + DE)(F + \bar{F}) + (\bar{C} + F)H \\
&= (\bar{A}B + C + DE) + H \\
&= X + H
\end{aligned}$$

Both solutions cost 17 literals.

4) Multiplexor/Adder/Subtractor

Consider the following 4-bit 2's complement adder/subtractor. When $OP = 0$, do an addition. Else (when $OP = 1$), do a subtraction. Specify how to implement each S_i and C_{i+1} using only the following 8:1 multiplexors.



5) Multiplexors and XOR gates

Given the following logic diagram, fill out the K-map.

Diagram corresponds to the following two-level logic.

$$\begin{aligned}
 F &= (A\bar{B} + B) \oplus C + AB + CD \\
 &= (A + B) \oplus C + AB + CD \\
 &= \bar{A}\bar{B}C + A\bar{C} + B\bar{C} + AB + CD
 \end{aligned}$$

