CSE166 – Image Processing – Final
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`http://www-cse.ucsd.edu/classes/fa10/cse166`
3-6pm Wed. Dec. 8, 2010.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 65. Good luck!

Part I: Fill in the Blank (1 pt. per blank, 20 pts. total).

1. The convolution of an $M \times N$ kernel with itself is of size ______________ .
2. If you design a classifier by looking at both the training and testing components of a dataset, you are at risk of ______________ .
3. The normal form of the line $ax+by+c=0$ is given by $\rho =$______________ and $\theta =$______________ .
4. If you fixate on an image of an American flag for 60 seconds and subsequently view a blank white screen, you will see the following color in place of red: ______________ .
5. In the most general case, the ______________ transform of a square is a parallelogram.
6. A covariance matrix proportional to `[1 1; 1 1]` has rank-______________ .
7. Huffman Coding is an example of a(n) ______________ length coding scheme.
8. A odd-symmetric Gabor filter is formed by computing the product of a(n) ______________ and a(n) ______________ .
9. The image enhancement operation that makes the probability density function of pixel brightnesses approximately uniform is called ______________ .
10. Given a set of vectors $\mathbf{x}_i, i = 1, \ldots, N$ with mean $\mathbf{m}_x$, the formula for the covariance matrix C_x is ______________ .
11. The lower bound in lossless image compression is set by the ______________ of the source.
12. The normalization constant for the N-tap binomial kernel $h(x) = a \begin{pmatrix} N-1 \\ x \end{pmatrix}$ for $x = 0, \ldots, N-1$ is given by $a =$______________ .
13. The axes of greatest and least inertia for a shape are given by the ______________ of the scatter matrix.
14. Given an RGB image, the ______________ channel representation is given by the three images $R-G$, $B-(R+G)/2$ and $(R+G+B)/3$.
15. The DC component is the lowest frequency component of the DFT; the highest is the ______________ component.
16. The Nth row of ______________ is produced by convolving the $(N-1)$st row with `[1 1]`.
17. An image with a highly peaked histogram has ______________ entropy than an image with a flat histogram .

18. If you forget to zero-pad when filtering in frequency domain, it can result in an problem in the resulting image known as ______________ .

19. The decision boundary of a minimum distance classifier between two classes in a 2D feature space is a(n) ______________ .

20. The inverse Fourier transform of a Gaussian is a(n) ______________ .

Part II: Written problems

1. (18 pts) This problem makes use of the following image sequence. Assume that the images are equal to zero outside their boundaries. *Hint*: the numbers have been chosen in this problem to be integer at each step.

$$I(x,y,t_o) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad I(x,y,t_o+1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 2 & 0 & 0 \\ 4 & 8 & 4 & 0 & 0 \\ 2 & 4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) If $I(x,y,t_o)$ is regarded as a filter (up to a constant), what type of kernel is it?

   (b) Compute ∇I at time t_o using the centered first difference approximation $\partial/\partial x \approx [0.5, 0, -0.5]$ and $\partial/\partial y \approx [0.5, 0, -0.5]'$.

   (c) Compute $\partial I/\partial t$ with the approximation $\partial/\partial t \approx [1, -1]$.

   (d) Solve for the windowed image second moment matrix of $I(x,y,t_o)$. What is its rank? Is this a 'good' interest point?

   (e) Suppose we want to compute the windowed image second moment matrix for all neighborhoods in a large image. Explain how this can be done efficiently, without nested for loops.

   (f) Specify A and $\mathbf{b}$ in the matrix-vector form of the optical flow equation $A\mathbf{u}+\mathbf{b} = \mathbf{0}$, where $\mathbf{u} = [u, v]^\top$ is the optical flow vector at the center pixel.

   (g) Solve for $\mathbf{u}$.

2. (5 pts.) Prove that the eigenvalues of a covariance matrix are non-negative. What is this property called?

3. (10 pts.) Recall that k-means is an iterative clustering algorithm.

   (a) What are the two steps of the k-means algorithm? What is a good way of initializing it?

   (b) Consider the following dataset of 2-dimensional feature vectors: `(3,3), (2,6), (2,5), (0,0), (0,3), (2,7)`. Suppose $k = 2$ and we initialize our means with the following two feature vectors: `(3,3), (2,6)`. Perform a single iteration of k-means on the above data. Compute J (the sum of squared error over all clusters) before and after performing the second step.

4. (12 pts) This problem makes use of the binary image displayed in Figure 1, in which black=1 and white=0. Note: in calculating the various quantities in this problem, round your answers to 2 significant figures.

   (a) Compute the coordinates of the centroid $\mathbf{m}$.

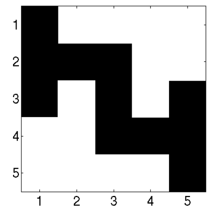

Figure 1: 5×5 binary image.

(b) Compute the scatter matrix C.

(c) Find the eigenvalues λ_1 and λ_2 of C.

(d) Find the angle ϕ of the principal eigenvector of C. Also write down the angle of the 2nd eigenvector. Express each answer in units of degrees.

(e) Letting $\mathbf{x}_k$ denote the original coordinates of the nonzero pixels, find the values of the rotation matrix R and translation vector $\mathbf{t}$ in the expression

$$\mathbf{x}'_k = R(\mathbf{x}_k + \mathbf{t}), \quad k = 1, 2, \ldots, 11$$

such that the set of transformed coordinates $\mathbf{x}'_k$ for $k = 1, 2, \ldots, 11$ is centered at the origin and has its principal axis aligned with the y axis.