CSE166 – Image Processing – Final
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`http://www-cse.ucsd.edu/classes/fa09/cse166`
11:30am-2:30pm Wed. Dec. 9, 2009.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 50. Good luck!

Part I: Fill in the Blank (1 pt. per blank, 20 pts. total).

1. The convolution of an $M \times N$ kernel with itself is of size ______________ .
2. The iterative algorithm we studied for clustering data in n dimensions into k groups is called ______________ .
3. The ______________ is a measure of the uncertainty of a random variable.
4. The operation comprising the steps "flip, shift, multiply and add" is called ______________ .
5. The Laplacian operator (without the Gaussian) is a ______________ -pass filter.
6. The inverse Fourier transform of a sinc function is a(n) ______________ .
7. Huffman Coding is of no use to compress an image with a(n) ______________ histogram.
8. We approximate $I_t(x, y, t)$ using the expression ______________ .
9. The ambiguity in ______________ as viewed in a small window is known as the aperture problem.
10. True or False: JPEG is recommended for compressing images of natural scenes. ______________
11. The axes of greatest and least ______________ for a shape are given by the eigenvectors of the scatter matrix.
12. The normal form of the line $ax+by+c = 0$ is given by $\rho =$______________ and $\theta =$______________ .
13. If you fixate on an image of an American flag for 60 seconds and subsequently view a blank white screen, you will see the following color in place of red: ______________ .
14. In the most general case, the ______________ transform of a square is a parallelogram.
15. A neighborhood of an image where an equal number of gradient vectors point up, down, left and right is an example of a rank-______________ neighborhood.
16. Huffman Coding is an example of a(n) ______________ length coding scheme.
17. When computing the covariance matrix given a set of vectors, you must first subtract off the ______________ from each vector.
18. The convolution of a box function with itself is a(n) ______________ .
19. The Hough transform was originally invented for detecting ______________ in images.

Part II: Written problems.

1. (12 pts) This problem makes use of the binary image displayed in Figure 1, in which black=1 and white=0. Note: in calculating the various quantities in this problem, round your answers to 2 significant figures.

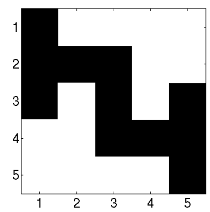

Figure 1: 5×5 binary image.

   (a) Compute the coordinates of the centroid $\mathbf{m}$.
   (b) Compute the scatter matrix C.
   (c) Find the eigenvalues λ_1 and λ_2 of C.
   (d) Find the angle ϕ of the principal eigenvector of C. Also write down the angle of the 2nd eigenvector. Express each answer in units of degrees.
   (e) Letting $\mathbf{x}_k$ denote the original coordinates of the nonzero pixels, find the values of the rotation matrix R and translation vector $\mathbf{t}$ in the expression

$$\mathbf{x}'_k = R(\mathbf{x}_k + \mathbf{t}), \quad k = 1, 2, \ldots, 11$$

   such that the set of transformed coordinates $\mathbf{x}'_k$ for $k = 1, 2, \ldots, 11$ is centered at the origin and has its principal axis aligned with the y axis.

2. (18 pts) This problem makes use of the following image sequence. Assume that the images are equal to zero outside their boundaries. *Hint*: the numbers have been chosen in this problem to be integer at each step.

$$I(x,y,t_o) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad I(x,y,t_o+1) = \begin{bmatrix} 0 & 2 & 4 & 2 & 0 \\ 0 & 4 & 8 & 4 & 0 \\ 0 & 2 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) If $I(x,y,t_o)$ is regarded as a filter (up to a constant), what type of kernel is it?
   (b) Compute ∇I at time t_o using the centered first difference approximation $\partial/\partial x \approx [0.5, 0, -0.5]$ and $\partial/\partial y \approx [0.5, 0, -0.5]'$.
   (c) Compute $\partial I/\partial t$ with the approximation $\partial/\partial t \approx [1, -1]$.
   (d) Solve for the windowed image second moment matrix of $I(x,y,t_o)$. What is its rank? Is this a 'good' interest point?
   (e) Specify A and $\mathbf{b}$ in the matrix-vector form of the optical flow equation $A\mathbf{u}+\mathbf{b} = \mathbf{0}$, where $\mathbf{u} = [u, v]^\top$ is the optical flow vector at the center pixel.
   (f) Solve for $\mathbf{u}$.