

CSE 103: Note Oct. 4 2007

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1 Random Variables

A random variable represent a measurement that can be repeated, but each time comes out differently. We *sample* the random variable

1. $X : \Omega \rightarrow R$
2. $\omega \in \Omega$ is an *outcome*
3. $A \subseteq \Omega$ is an *event*

Probabilities are assigned to events.

1. $P(\Omega) = 1$
2. $1 \geq P(A) \geq 0$
3. if A and B are disjoint, then $P(A \cup B) = P(A) + P(B)$

2 CDFs

Definition. $F_X(x) := P(X \leq x)$ Corresponds to a non decreasing function going from 0 at $-\infty$ to 1 at $+\infty$.

3 Distributions

Two flavors of distributions:

1. Discrete distributions : defined by probability on single outcomes
2. Continuous distributions : defined by a probability density function

$$p(x) \geq 0 \text{ s.t. } \int_{-\infty}^{\infty} p(x)dx = 1$$

Example of a mixture of discrete and continuous distributions:

1. Speeds of randomly sampled cars.
2. The wavelength of photons coming from a bright star:
 - continuous: hot body emission (lightbulb)
 - discrete: photons from specific energy releases (fluorescent bulb).

Jumps in the CDF correspond to elements with non-zero probability. Slopes correspond to densities.

4 Independence

1. Independent events $P(A \cap B) = P(A)P(B)$
2. Independent RV: $\forall E_1, E_2 \subset R$

$$P(X \in E_1 \text{ and } Y \in E_2) = P(x \in E_1)P(y \in E_2)$$

Special cases:

- Independent discrete RV:

$$\forall x, y \ P(X = x \text{ and } Y = y) = P(X = x)P(Y = y)$$

[binary is a special case in which just one value pair needs to be checked]

- Independent contin. RV: $\forall x_1 < x_2, y_1 \leq y_2 :$

$$P(x_1 \leq X \leq x_2 \text{ and } y_1 \leq Y \leq y_2) = P(x_1 \leq X \leq x_2)P(y_1 \leq Y \leq y_2)$$

5 Expected Value

$E(X) := \sum_x xp(x) = \int x dP(x)$ (Last integral is a Lebesgue integral, which is more powerful than the Riemann Integral)

Properties of expectation

- linearity: $E(aX + bY) = aE(X) + bE(Y)$
- $E(a) = a$
- $\mu = E(X)$ corresponds to the center of mass
- If X and Y are independent $E(XY) = E(X)E(Y)$

6 Variance

- $Var(X) = E((X - \mu)^2) = E(X^2 - 2\mu X + \mu^2) = E(X^2) - E(X)^2$
- $Var(2X) = E(4X^2) - E(2X)^2 = 4Var(X)$

Therefore we look at std-dev = $\sigma(X) = \sqrt{Var(X)}$ as a measure of the spread of the distribution.

Assume $X_1, X_2, \dots, X_n$ are all independent and consider the variance of $\sum_{i=1}^n X_i$

$$E((\sum X_i)^2) = nE(X_i^2) = nVar(X)$$

Therefore std-dev($\sum_{i=1}^n X_i$) scales like $\sqrt{n}$ remember this!!!