# kstest

One-sample Kolmogorov-Smirnov test

## Syntax

```
H = kstest(X)
H = kstest(X,cdf)
H = kstest(X,cdf,alpha)
H = kstest(X,cdf,alpha,tail)

[H,P] = kstest(...)

[H,P,KSSTAT,] = kstest(...)
[H,P,KSSTAT,CV] = kstest(...)
```

## Description

`H = kstest(X)` performs a Kolmogorov-Smirnov test to compare the values in the data vector `X` with a standard normal distribution (that is, a normal distribution having mean 0 and variance 1). The null hypothesis for the Kolmogorov-Smirnov test is that `X` has a standard normal distribution. The alternative hypothesis that `X` does not have that distribution. The result `H` is `1` if you can reject the hypothesis that `X` has a standard normal distribution, or `0` if you cannot reject that hypothesis. You reject the hypothesis if the test is significant at the 5% level.

For each potential value *x*, the Kolmogorov-Smirnov test compares the proportion of values less than *x* with the expected number predicted by the standard normal distribution. The `kstest` function uses the maximum difference over all *x* values is its test statistic. Mathematically, this can be written as

$$\max(|F(x) - G(x)|)$$

where $F(x)$ is the proportion of `X` values less than or equal to *x* and $G(x)$ is the standard normal cumulative distribution function evaluated at *x*.

`H = kstest(X,cdf)` compares the distribution of `X` to the hypothesized continuous distribution defined by the two-column matrix `cdf`. Column one contains a set of possible *x* values, and column two contains the corresponding hypothesized cumulative distribution function values $G(x)$. If possible, you should define cdf so that column one contains the values in X. If there are values in X not found in column one of cdf, kstest will approximate $G(X)$ by interpolation. All values in X must lie in the interval between the smallest and largest values in the first column of cdf. If the second argument is empty (cdf = []), kstest uses the standard normal distribution as if there were no second argument.

The Kolmogorov-Smirnov test requires that cdf be predetermined. It is not accurate if cdf is estimated from the data. To test X against a normal distribution without specifying the parameters, use lillietest instead.

H = kstest(X,cdf,alpha) specifies the significance level alpha for the test. The default is 0.05.

H = kstest(X,cdf,alpha,*tail*) specifies the type of test in the string *tail*. *tail* can have one of the following values:

- 'unequal'
- 'larger'
- 'smaller'

The tests specified by these values are described in Tests Specified by tail.

[H,P,KSSTAT,CV] = kstest(X,cdf,alpha,tail) also returns the observed p-value P, the observed Kolmogorov-Smirnov statistic KSSTAT, and the cutoff value CV for determining if KSSTAT is significant. If the return value of CV is NaN, then kstest determined the significance calculating a p-value according to an asymptotic formula rather than by comparing KSSTAT to a critical value.

### Tests Specified by tail

Let S(*x*) be the empirical c.d.f. estimated from the sample vector X, let F(*x*) be the corresponding true (but unknown) population c.d.f., and let CDF be the known input c.d.f. specified under the null hypothesis. The one-sample Kolmogorov-Smirnov test tests the null hypothesis that F(*x*) = CDF for all *x* against the alternative specified by one of the following possible values of tail:

| tail | Alternative Hypothesis | Test Statistic |
|---|---|---|
| 'unequal' | F(*x*) does not equal CDF (two-sided test) | max\|S(*x*) - CDF\| |
| 'larger' | F(*x*) > CDF (one-sided test) | max[S(*x*) - CDF] |

| | | |
|---|---|---|
| 'smaller' | F(*x*) < CDF (one-sided test) | max[S(*x*) - CDF] |

## Examples

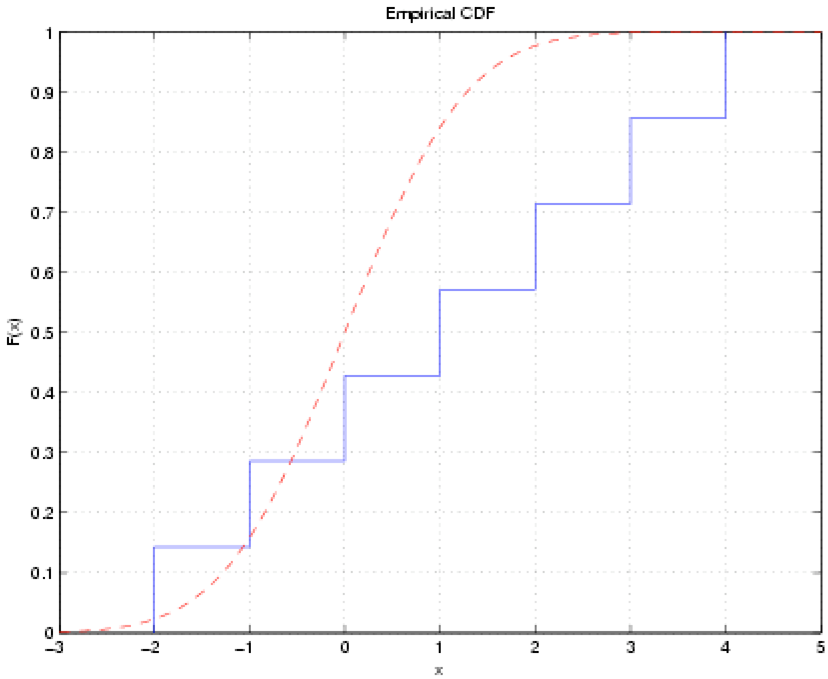
**Example 1.** Let's generate some evenly spaced numbers and perform a Kolmogorov-Smirnov test to see how well they fit to a standard normal distribution:

```
x = -2:1:4
x =
  -2  -1   0   1   2   3   4

[h,p,k,c] = kstest(x,[],0.05,0)

h =
   0
p =
   0.13632
k =
   0.41277
c =
   0.48342
```

You cannot reject the null hypothesis that the values come from a standard normal distribution. Although intuitively it seems that these evenly-spaced integers could not follow a normal distribution, this example illustrates the difficulty in testing normality in small samples.

To understand the test, it is helpful to generate an empirical cumulative distribution plot and overlay the theoretical normal distribution.

```
xx = -3:.1:5;
cdfplot(x)
hold on
plot(xx,normcdf(xx),'r-')
```




The Kolmogorov–Smirnov test statistic is the maximum difference between these curves. It appears that this maximum of 0.41277 occurs as the data approaches `x = 1.0` from below. You can see that the empirical curve has the value 3/7 here, and you can easily verify that the difference between the curves is 0.41277.

```
normcdf(1) - 3/7
ans =
   0.41277
```

You can also perform a one-sided test. Setting $tail$ = `-1`indicates that the alternative is $F < G$, so the test statistic counts only points where this inequality is true.

```
[h,p,k] = kstest(x, [], .05, -1)

h =
   0

p =
   0.068181

k =
   0.41277
```

The test statistic is the same as before because in fact $F < G$ at `x = 1.0`. However, the p-value is smaller for the one-sided test. If you carry out the other one-sided test, you see that the test statistic changes, and is the difference between the two curves near x = -1.0.

```
[h,p,k] = kstest(x,[],0.05,1)

h =
   0

p =
   0.77533

k =
   0.12706

2/7 - normcdf(-1)

ans =
   0.12706
```

**Example 2.** Now let's generate random numbers from a Weibull distribution, and test against that Weibull distribution and an exponential distribution.

```
x = wblrnd(1, 2, 100, 1);
kstest(x, [x wblcdf(x, 1, 2)])

ans =
   0

kstest(x, [x expcdf(x, 1)])

ans =
   1
```

## See Also

kstest2, lillietest