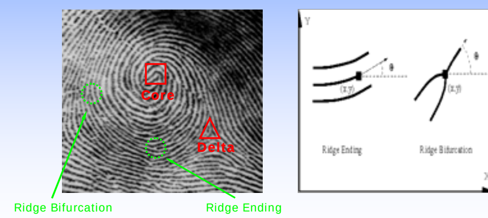
Finger Print Recognition
(RANSAC)
and back to Face Rec

Biometrics
CSE 190-a
Lecture 15

Fingerprint Representation

- Local ridge characteristics (minutiae): ridge ending and ridge bifurcation
- Singular points: Discontinuity in ridge orientation



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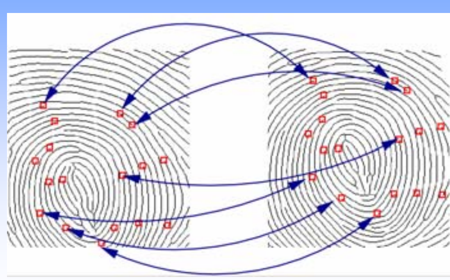
Minutiae Correspondences



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Minutiae Matching

- Point pattern matching problem
- Let $P = \{(x_1^P, y_1^P, \theta_1^P), \ldots, (x_M^P, y_M^P, \theta_M^P)\}$ be the set of M minutiae in the template image
- Let $Q = \{(x_1^Q, y_1^Q, \theta_1^Q), \ldots, (x_N^Q, y_N^Q, \theta_N^Q)\}$ be the set of N minutiae in the input image
- Find the number of corresponding minutia pairs between P and Q and compare it against a threshold

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Stages of Minutiae-based Verification

- Extract Minutiae using corner detection
- Characterize (label) Minutiae
- Transformations between fingerprint images
- RANSAC

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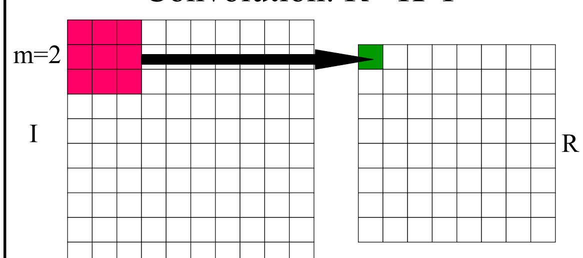
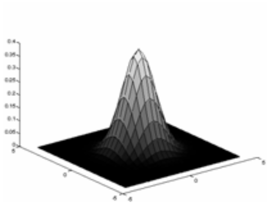
Convolution: R= K*I



Kernel size
is m+1 by m+1

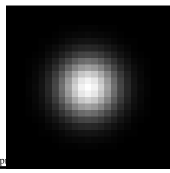
$$R(i,j) = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} K(h,k) I(i-h, j-k)$$

An Isotropic Gaussian



- The picture shows a smoothing kernel proportional to

$$\exp\left(-\left(\frac{x^2+y^2}{2\sigma^2}\right)\right)$$



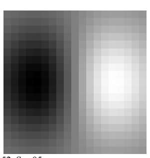
(which is a reasonable model of a circularly symmetric fuzzy blob)

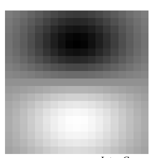
On numerical derivatives

Convolve with
First Derivative: [-1 0 1]
Second Derivative: [-1 2 -1]
First Derivative in Y Direction:$[-1\ 0\ 1]^T$

Smoothing and Differentiation

- Need two derivatives, in x and y direction.
- Filter with Gaussian and then compute gradient, or
- Use a derivative of Gaussian filter
  - because differentiation is convolution, and convolution is associative





Formula for Finding Corners

Let $I_x = \frac{\partial I}{\partial x}, and\ I_y = \frac{\partial I}{\partial y}$

Sum over a small region, the hypothetical corner

Gradient with respect to x, times gradient with respect to y

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

Matrix is symmetric

WHY THIS?

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First, consider case where:

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means all gradients in neighborhood are:

(k,0) or (0, c) or (0, 0) (or off-diagonals cancel).

What is region like if:

1. $\lambda_1 = 0?$
2. $\lambda_2 = 0?$
3. $\lambda_1 = 0$ and $\lambda_2 = 0?$
4. $\lambda_1 > 0$ and $\lambda_2 > 0?$

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General Case:

From Linear Algebra, it follows that

$$C = R^{-1}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

since C is symmetric. So every case is like the one on the last slide.

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So, to detect corners

- Filter image.
- Compute the gradient everywhere.
- We construct C in a window of some size.

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

- Find eigenvalus λ_1 and λ_2.
- If λ_1 and λ_2 are both big, we have a corner.
  1. Let $e(u,v) = \min(\lambda_1(u,v), \lambda_2(u,v))$
  2. (u,v) is a corner local maximum of $e(u,v)$ and $e(u,v) > \tau$

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Labeling Minutiae

- The corners found by above procedure include Minutiae, but also other "spurious" points.
- Label each corner as:
  - Core
  - Delta
  - Ridge bifurcations
  - Ridge endings
  - Other
- Build 5-class classifier given training data
- Is it built over rotations/scales? Other?

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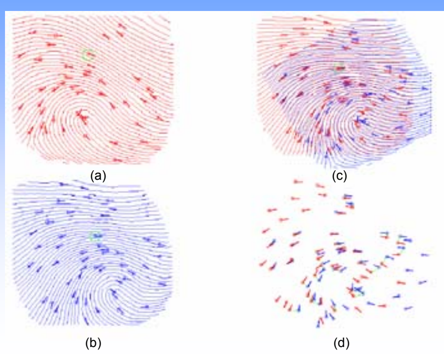
Minutiae Matching Result



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Allowable transformations

- Translation
- Rigid Transformations
- Affine Transformations
- Warps (e.g., thin plate splines)

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RANSAC

Slides shamelessly taken from
Frank Dellaert and Marc Pollefeys
and modified

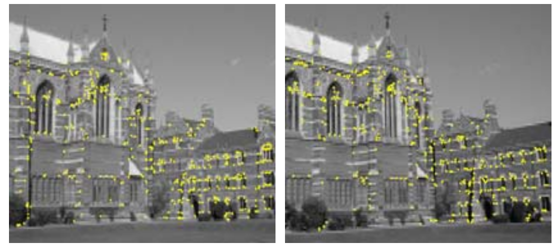
Motivation

- Estimating motion models
- Typically: points in two images
- Candidates:
  - Translation
  - Affine
  - Homography

Mosaicking: Homography

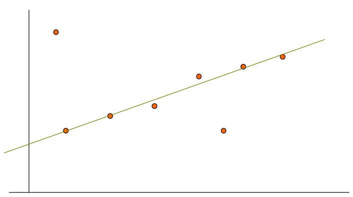
www.cs.cmu.edu/~dellaert/mosaicking

Fundamental Matrix



Simpler Example

- Fitting a straight line



- Inliers
- Outliers

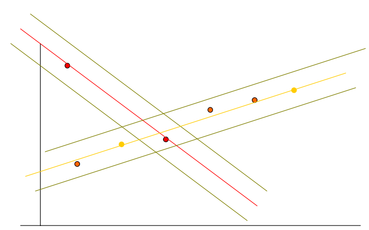
Discard Outliers

- No point with d>t
- RANSAC:
  - RANdom SAmple Consensus
  - Fischler & Bolles 1981
  - Copes with a large proportion of outliers

Main Idea

- Select 2 points at random
- Fit a line
- “Support” = number of inliers
- Line with most inliers wins

Why will this work ?



Best Line has most support

- More support -> better fit

RANSAC

<u>Objective</u>

Robust fit of model to data set S which contains outliers

<u>Algorithm</u>

(i) Randomly select a sample of *s* data points from S and instantiate the model from this subset.

(ii) Determine the set of data points S_i which are within a distance threshold *t* of the model. The set S_i is the **consensus set** of samples and defines the inliers of S.

(iii) If the subset of S_i is greater than some threshold *T*, re-estimate the model using all the points in S_i and terminate

(iv) If the size of S_i is less than *T*, select a new subset and repeat the above.

(v) After *N* trials the largest consensus set S_i is selected, and the model is re-estimated using all the points in the subset S_i

Distance threshold

Choose *t* so probability for inlier is α (e.g. 0.95)

- Often empirically
- Zero-mean Gaussian noise σ then $d_\perp^2$ follows χ_m^2 distribution with *m*=codimension of model

(dimension+codimension=dimension space)

Codimension	Model	t^2
1	l,F	3.84σ²
2	H,P	5.99σ²
3	T	7.81σ²

How many samples?

Choose *N* so that, with probability *p*, at least one random sample is free from outliers. e.g. *p*=0.99

$$\left(1-(1-e)^s\right)^N = 1-p$$

$$N = \log(1-p)/\log\left(1-(1-e)^s\right)$$

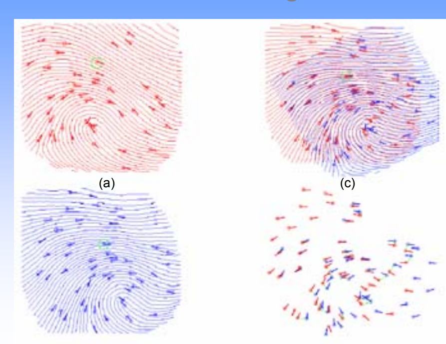
	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Acceptable consensus set?

- Typically, terminate when inlier ratio reaches expected ratio of inliers

$$T = (1-e)n$$

Minutiae Matching Result



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Minutiae Extraction Failure

A

B

True Minutiae Matches: A1→B3, A18→B9, A19→B7

A1, B9 and B7 were detected, but the associated ridges were not detected because they are close to the boundary

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Alignment Failure

A

B

True Minutiae Matches: A7→B9, A8→B8, A4→B1

A7→B9 and A8→B8 pairs have ridge points; however, there exists a false alignment that results in more than three matches

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Matching Failure

A

B

No. of matching minutiae identified by the matcher = 10

No. of minutiae in A = 38; No. of minutiae in B = 34

Spurious minutiae and large deformation leads to small score

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